

A Method for Solving Distributed Service Allocation Problems *

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ABSTRACT

We present a method for solving service allocation problems in which a set of services must be allocated to a set of agents so as to maximize a global utility. The method is completely distributed so it can scale to any number of services without degradation. We first formalize the service allocation problem and then present a simple hill-climbing, a global hill-climbing, and a bidding-protocol algorithm for solving it. We analyze the expected performance of these algorithms as a function of various problem parameters such as the branching factor and the number of agents. Finally, we use the sensor allocation problem, an instance of a service allocation problem, to show the bidding protocol at work. The simulations also show that phase transition on the expected quality of the solution exists as the amount of communication between agents increases.

1. INTRODUCTION

The problem of dynamically allocating services to a changing set of consumers arises in many applications. For example, in an e-commerce system, the service providers are always trying to determine which service to provide to whom, and at what price [3]; in an automated manufacturing for mass customization scenario, agents must decide which services will be more popular/profitable [1]; and in a dynamic sensor allocation problem, a set of sensors in a field must decide which area to cover, if any, while preserving their resources.

While these problems might not seem related, they are instances of a more general service allocation problem in which a finite set of resources must be allocated by a set of autonomous agents so as to maximize some global measure of utility. A general approach to solving these types of prob-

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lems has been used in many successful systems. The approach involves three general steps:

1. Assign each resource that needs to be preserved to an agent responsible for managing the resource.
2. Assign each goal of the problem domain to an agent responsible for achieving it. Achieving these goals requires the consumption of resources.
3. Have each agent take actions so as to maximize its own utility, but implement a coordination algorithm that encourages agents to take actions that also maximize the global utility.

In this paper we formalize this general approach by casting the problem as a search in a global *fitness landscape* which is defined as the sum of the agents' utilities. We show how the choice of a coordination/communication protocol disseminates information, which in turn "smooths" the global utility landscape. This smooth global utility landscape allows the agents to easily find the global optimum by simply making selfish decisions to maximize their individual utility.

We also present experiments that pinpoint the location of a *phase transition* in the time it takes for the agents to find the optimal allocation. The transition can be seen when the amount of communication allowed among agents is manipulated. It exists because communication allows the agents to align their individual landscapes with the global landscape. At some amount of communication, the alignment between these landscapes is good enough to allow the agents to find the global optimum, but less communication drives the agents into a random behavior from which the system cannot recuperate.

1.1 Task Allocation

The service allocation problem we discuss in this paper is a superset of the well known task allocation problem [8, chapter 5.7]. A task allocation problem is defined by a set of tasks that must be allocated among a set of agents. Each agent has a cost associated with each subset of tasks, which represents the cost the agent would incur if it had to perform those tasks. Coordination protocols are designed to allow agents to trade tasks so that the globally optimal

allocation—the one that minimizes the sum of all the individual agent costs—is reached as soon as possible. It has been shown that this globally optimal allocation can be reached if the agents use the contract-net protocol [7] with OCSM contracts [6]. These OCSM contracts make it possible for the system to transition from any allocation to any other allocation in one step. As such, a simple hill-climbing search is guaranteed to eventually reach the global optimum.

In this paper we consider the service allocation problem, which is a superset of the task allocation because it allows for more than one agent to service a “task”. The service allocation problem we study also has the characteristic that every allocation cannot be reached from every other allocation in one step.

1.2 Service Allocation

In a service allocation problem there are a set of services, offered by service agents, and a set of consumers who use those services. A server can provide any one of a number of services and some consumers will benefit from that service without depleting it. A server agent incurs a cost when providing a service and can choose not to provide any service.

For example, a server could be an agent that sets up a website with information about cats. All the consumer agents with interests in cats will benefit from this service, but those with other interests will not benefit. Since each server can provide, at most, one service, the problem is to find the allocation of services that maximizes the sum of all the agents’ utilities, that is, an allocation that maximizes the global utility.

1.2.1 Sensor Allocation

Another instance of the service allocation problem is the sensor allocation problem, which we will use as an example throughout this paper. In the sensor allocation problem we have a number of sensors placed in fixed positions in a two-dimensional space. Each sensor has a limited viewing angle and distance but can point in any one of a number of directions. For example, a sensor might have a viewing angle of 120 degrees, viewing distance of 3 feet, and be able to look in three directions, each one 120 degrees apart from the others. That is, it can “look” in any one of three directions. On each direction it can see everything that is in the 120 degree and 3 feet long view cone. Each time a sensor looks in a particular direction it uses energy.

There are also targets that move around in the field. The goal is for the sensors to detect and track all the targets in the field. However, in order to determine the location of a target, two or more sensors have to look at it at the same time. We also wish to minimize the amount of energy spent by the sensors.

We consider the sensor agents as being able to provide three services, one for each sector, but only one at a time. We consider the target agents as consuming the services of the sensors.

2. A FORMAL MODEL FOR SERVICE ALLOCATION

We define a service allocation problem SA as a tuple $SA = \{C, S\}$ where C is the set of **consumer** agents $C = \{c_1, c_2, \dots, c_{|C|}\}$, and c_i has only one possible state, $c_i = 0$. The set of **service** agents is $S = \{s_1, s_2, \dots, s_{|S|}\}$ and the value of s_i is the value of that service. For the sensor domain in which a sensor can observe any one of three 120-degree sectors or be turned off, we have $s_i \in \{0, 1, 2, \text{off}\}$. An **allocation** is an assignment of states to the services (since the consumers have only one possible state we can ignore them). A particular allocation is denoted by $a = \{s_1, s_2, \dots, s_{|S|}\}$, where the s_i have some value taken from the domain of service states, and $a \in A$, where A is the set of all possible allocations. That is, an allocation tells us the state of all agents (since consumers have only one state they can be omitted).

Each agent also has a **utility** function. The utility that an agent receives depends on the current allocation a , where we let $a(s)$ be the state of service agent s under a . The agent’s utilities will depend on their state and the state of other agents. For example, in the sensor problem we define the utility of sensor s as $U_s(a)$, where

$$U_s(a) = \begin{cases} 0 & \text{if } a(s) = \text{off} \\ -K_1 & \text{otherwise.} \end{cases} \quad (1)$$

That is, a sensor receives no utility when it is off and must pay a penalty of $-K_1$ when it is running.

The targets are the consumers, and each target’s utility is defined as

$$U_c(a) = \begin{cases} 0 & \text{if } f_c(a) = 0 \\ K_2 & \text{if } f_c(a) = 1 \\ K_2 + n - 2 & \text{if } f_c(a) = n \end{cases} \quad (2)$$

where

$$f_c(a) = \text{number of sensors } s \text{ that see } c \text{ given their state } a(c). \quad (3)$$

Finally, given the individual agent utilities, we define the **global utility** $GU(a)$ as the sum of the individual agents’ utilities:

$$GU(a) = \sum_{c \in C} U_c(a) + \sum_{s \in S} U_s(a). \quad (4)$$

The service allocation problem is to find the allocation a that maximizes $GU(a)$. In the sensor problem, there are $4^{|S|}$ possible allocations, which would make a simple generate-and-test approach take exponential amounts of time. We wish to find the global optimum much faster than that.

2.1 Search Algorithms

Our goal is to design an interaction protocol whereby an allocation a that maximizes the global utility $GU(a)$ is reached in a small number of steps. In each step of our protocol one of the agents will change its state or send a message to another agent. The messages might contain the state or utilities of other agents. We assume that the agents do not have direct access to the other agents’ states or utility values.

The simplest algorithm we can envision involves having each consumer, at each time, changing the state of a randomly chosen service agent so as to increase the consumer’s own

utility. That is, a consumer c will change the current allocation a into a' by changing the state of some sensor s such that $U_c(a') > U_c(a)$. If the sensor's state cannot be changed so as to increase the utility, then the consumer does nothing. In the sensor domain this amounts to a target picking a sensor and changing its state so that the sensor can see the target. We refer to this algorithm as **individual hill-climbing**.

The individual hill-climbing algorithm is simple to implement and the only communication needed is between the consumer and the chosen server. This simple algorithm makes every consumer agent increase its individual utility at each turn. However, the new allocation a' might result in a lower global utility, since a' might reduce the utility of several other agents. Therefore, it does not guarantee that an optimal allocation will be eventually reached.

Another approach is for each agent to change state so as to increase the global utility. We call this a **global hill-climbing** algorithm. In order to implement this algorithm, an agent would need to know how the proposed state change affects the global utility as well as the states of all the other agents. That is, it would need to be able to determine $GU(a')$ which requires it to know the state of all the agents in a' as well as the utility functions of every other agent, as per the definition of global utility (4). In order for an agent to know the state of others, it would need to somehow communicate with all other agents. If the system implements a global broadcasting method then we would need for each agent to broadcast its state at each time. If the system uses more specialized communications such as point-to-point, limited broadcasting, etc., then more messages will be needed.

Any protocol that implements the global hill-climbing algorithm will reach a locally optimal allocation in the global utility. This is because it is always true that, for a new allocation a' and old allocation a , $GU(a') \geq GU(a)$. Whether or not this local optimum is also a global optimum will depend on the ruggedness of the global utility landscape. That is, if it consists of one smooth peak then it is likely that any local optimum is the global optimum. On the other hand, if the landscape is very rugged then there are likely many local peaks. Studies in NK landscapes [2] tell us that smoother landscapes result when an agent's utility depends on the state of smaller number of other agents.

Global hill-climbing is better than individual hill-climbing since it guarantees that we will find a local optima. However, it requires agents to know each others' utility function and to constantly communicate their state. Such large amount of communication is often undesirable in multiagent systems. We need a better way to find the global optimum.

One way of correlating the individual landscapes to the global utility landscape is with the use of a **bidding protocol** in which each consumer agent tells each service the marginal utility the consumer would receive if the service switched its state to so as to maximize the consumer's utility. The service agent can then choose to provide the service with the highest aggregate demand. Since the service is picking the value that maximizes the utility of everyone involved

(all the consumers and the service) without decreasing the utility of anyone else (the other services) this protocol is guaranteed to never decrease the global utility. This bidding protocol is a simplified version of the contract-net [7] protocol in that it does not require contractors to send requests for bids.

However, in order for a consumer to determine the marginal utility it will receive from one sensor changing state, it still needs to know the state of all the other sensors. This means that a complete implementation of this protocol will still require a lot of communication (namely, the same amount as in global hill-climbing). We can reduce this number of messages by allowing agents to communicate with only a subset of the other agents and making their decisions based on only this subset of information. That is, instead of all services telling each consumer their state, a consumer could receive state information from only a subset of the services and make its decision based on this (assuming that the services chosen are representative of the whole). This strategy shows a lot of promise but its performance can only be evaluated on an instance-by-instance basis. We explore this strategy experimentally in Section 3 using the sensor domain.

2.1.1 Theoretical Time Bounds of Global Hill-Climbing

Since we now know that global hill-climbing will always reach a local optimum, the next questions we must answer are:

1. How many local optima are there?
2. What is the probability that a local optimum is the global optimum?
3. How long does it take, on average, to reach a local optimum?

Let a be the current allocation and a' be a neighboring allocation. We know that a is a local optimum if

$$\forall_{a' \in N(a)} GU(a) > GU(a') \quad (5)$$

where

$$N(a) = \{x \mid x \text{ is a Neighbor of } a\}. \quad (6)$$

We define a *Neighbor* allocation as an allocation where one, and only one, agent has a different state.

The probability that some allocation a is a local optimum is simply the probability that (5) is true. If the utility of all pairs of neighbors is not correlated, then this probability is

$$\Pr[\forall_{a' \in N(a)} GU(a) > GU(a')] = \Pr[GU(a) > GU(a')]^b, \quad (7)$$

where b is the **branching factor**. In the sensor problem $b = 3 \cdot |S|$ where S is the set of all sensors. That is, since each sensor can be in any of four states it will have three neighbors from each state. In some systems it is safe to assume that the global utilities of a 's neighbors are independent. However, most systems show some degree of correlation.

Now we need to calculate the $\Pr[GU(a) > GU(a')]$, that is, the probability that some allocation a has a greater global utility than its neighbor a' , for all a and a' . This could

be calculated via an exhaustive enumeration of all possible allocations. However, often we can find the expected value of this probability.

For example, in the sensor problem each sensor has four possible states. If a sensor changes its state from sector x to sector y the utility of the target agents covered by x will decrease while the utility of those in y will increase. If we assume that, on average, the targets are evenly spaced on the field, then the global utilities for both of these are expected to be the same. That is, the expected probability that the global utility of one allocation is bigger than the other is $1/2$.

If, on the other hand, a sensor changes state from “off” to a sector, or from a sector to “off,” the global utility is expected to decrease and increase, respectively. However, there are an equal number of opportunities to go from “off” to “on” and vice-versa. Therefore, we can also expect that for these cases the probability that the global utility of one allocation is bigger than the other is $1/2$.

Based on these approximations, we can declare that for the sensor problem

$$\Pr[\forall_{a' \in N(a)} GU(a) > GU(a')] = \frac{1}{2^b} = \lambda. \quad (8)$$

If we assume an even distribution of local optima, the total number of local optima is simply the product of the total number of allocations times the probability that each one is a local optimum. That is,

$$\text{Total number of local optima} = \lambda|A| \quad (9)$$

For the sensor problem, $\lambda = 1/2^b$, $b = 3 \cdot |S|$ and $|A| = b^{|S|}$, so the expected number of local optima is $b^{|S|}/2^{3|S|}$.

$$\Pr[\text{a local optimum is also global}] = \frac{1}{\lambda|A|} = \frac{1}{2^b}. \quad (10)$$

We can find the expected time the algorithm will take to reach a local optimum by determining the maximum number of steps from every allocation to the nearest local optimum. This gives us an upper bound on the number of steps needed to reach the nearest local optimum using global hill-climbing. Notice that, under either individual hill-climbing or the bidding protocol it is possible that the local optimum is not reached, or is reached after more steps, since these algorithms can take steps that lower the global utility.

In order to find the expected number of steps to reach a local optimum, we start at any one of the local optima and then traverse all possible links at each depth d until all possible allocations have been visited. This occurs when

$$\lambda \cdot |A| \cdot b^d > |A|. \quad (11)$$

Solving for d , and remembering that $\lambda = 1/2^b$, we get

$$d > b \log_b 2. \quad (12)$$

The expected worst-case distance from any point to the nearest local optimum is, therefore, $b \log_b 2$ (this number only makes sense for $b \geq 2$ since smaller number of neighbors do not form a searchable space). That is, the number of steps to reach the nearest local optima in the sensor domain is proportional to the branching factor b , which is equal to $3 \cdot |S|$. We can expect search time to increase linearly with the number of sensors in the field.

3. SIMULATIONS

While the theoretical results above give us some bounds on the number of iterations before the system is expected to converge to a local optimum, the bounds are rather loose and do not tell us much about the dynamics of the executing system. Also, we cannot show mathematically how changes in the amount of communication change the search. Therefore, we have implemented a service allocation simulator to answer these questions. It simulates the sensor allocation domain described in the introduction.

The simulator is written in Java and the source code is available¹. It gathers and analyzes data from any desired number of runs. The program can analyze the behavior of any number of target and sensor agents on a two-dimensional space, and the agents can be given any desired utility function. The program is limited to static targets. That is, it only considers the one-shot service allocation problem. Each new allocation is completely independent of any previous one.

In the tests we performed, each run has eight sensors and eight targets, all of which are randomly placed on a two-dimensional grid. Each sensor can only point in one of three directions or sectors. These three sectors are the same for all sensors (specifically, the first sector is from 0 to 120 degrees, the second one from 120 to 240, and the third one from 240 to 360). The sensor’s utility is given by (1), while the targets use (2).

During a run, each of the targets periodically sends a bid to a number of sensors asking them to turn to the sector that faces the target. We set the bid amount to a fixed number for these tests. Periodically, the sensors count the number of bids they have received for each sector and turn their detector (such as a radar) to face the sector with the highest aggregate demand.

We vary the number of sensors to which the targets send their bids. For example, if the all the targets send their bids to all the sensors, then the sensors would always set their sector to be the one with the most targets. This particular service allocation should, usually, be the best. However, it might not always be the optimal solution. For example, if seven targets are clustered together and the eighth is on another part of the field, it would be better if six sensor agents pointed towards the cluster of targets while the remaining two sensor agents pointed towards the stray target rather than having all sensor agents point towards the cluster of targets.

These simulations explore the ruggedness of the system’s global utility landscape and the dynamics of the agents’ ex-

¹<http://blind.review>

ploration of this landscape. If the agents were to always converge on a local (non-global) optimum then we would deduce that this problem domain has a very rugged utility landscape. On the other hand, if they usually manage to reach the global optimum then we could deduce a smooth utility landscape.

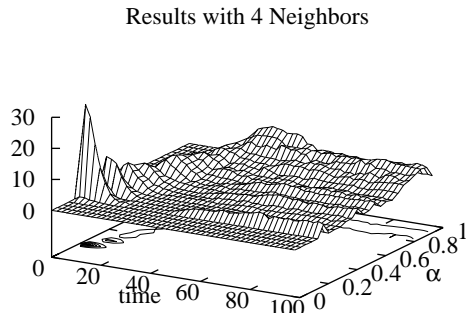


Figure 2: The transitional case occurs when the target communicates with four sensors.

4. TEST RESULTS

In each of our tests we set the number of agents that each target will send its bid to, that is, the number of neighbors, to a fixed number. Given this fixed number of neighbors, we then generated 100 random placements of agents on the field and ran our bidding algorithm 10 times on each of those placements. Finally, we plotted the average solution quality, over the 10 runs, as a function of time for each of the 100 different placements. The solution quality is given by the ratio

$$\alpha = \frac{\text{Current Utility}}{\text{Globally Optimal Utility}}, \quad (13)$$

so if $\alpha = 1$, then it means that the run has reached the global optimum. Since the number of agents is small, we were able to calculate the global optimum using a brute-force method.

In our tests there were always seven target agents and seven sensor agents. We varied the number of neighbors from 1 to 7. If the target can only communicate with one other sensor, the sensors will likely have very little information for making their decision, while if all targets communicate with all seven sensors, then each sensor will generally be able to point to the sector with the most targets. However, because these decisions are made in an asynchronous manner, it is possible that some sensor will sometimes not receive all the bids before it has to make a decision.

The results from our experiments are shown in Figure 1 where we can see that there is a transition in the system’s performance as the number of neighbors goes from three to five. That is, if the targets only send their bids to three sensors then it is almost certain that the system will stay in a configuration that has a very low global utility. However, if the targets send their bids to five sensors, then it is almost guaranteed (98% of the time) that the system will reach the globally optimal allocation. This is a huge difference in

terms of the performance. We also notice in Figure 2 that for four neighbors there is a fairly even distribution in the utility of the final allocation.

5. RELATED WORK

There is ongoing work in the field of complexity that attempts to study the dynamics of complex adaptive systems [2]. Our approach is based on ideas borrowed from the use of NK landscapes for the analysis of co-evolving systems. As such, we are using some of the results from that field. However, complexity theory is more concerned with explaining the dynamic behavior of existing systems, while we are more concerned with the engineering of multiagent systems for distributed service allocation.

The Collective Intelligence (COIN) framework [9] shares many of the same goals of our research. They start with a global utility function from which they derive the rewards functions for each agent. The agents are assumed to use some form of reinforcement learning. They show that the global utility is maximized when using their prescribed reward functions. They do not, however, consider how agent communication might affect the individual agent’s utility landscape.

The task allocation problem has been studied in [5], but the service allocation problem we present in this paper has received very little attention. There is also work being done on the analysis of the dynamics of multiagent systems for other domains such as e-commerce [3] and automated manufacturing [4]. It is possible that extensions to our approach will shed some light into the dynamics of these domains.

6. CONCLUSIONS

We have formalized the service allocation problem and examined a general approach to solving problems of this type. The approach involves the use of utility-maximizing agents that represent the resources and the services. A simple form of bidding is used for communication. An analysis of this approach reveals that it implements a form of distributed hill-climbing, where each agent climbs its own utility landscape and not the global utility landscape. However, we showed that increasing the amount of communication among the agents forces each individual agent’s landscape to become increasingly correlated to the global landscape.

These theoretical results were then verified in our implementation of a sensor allocation problem—an instance of a service allocation problem. Furthermore, the simulations allowed us to determine the location of a phase transition in the amount of communication needed for the system to consistently arrive at the globally optimal service allocation.

More generally, we have shown how a service allocation problem can be viewed as a distributed search by multiple agent over multiple landscapes. We also showed how the correlation between the global utility landscape and the individual agent’s utility landscape depends on the amount and type of inter-agent communication. Specifically, we have shown that increased communications leads to a higher correlation between the global and individual utility landscapes, which increases the probability that the global optimum will be reached. Of course, the success of the search still depends on the connectivity of the search space, which

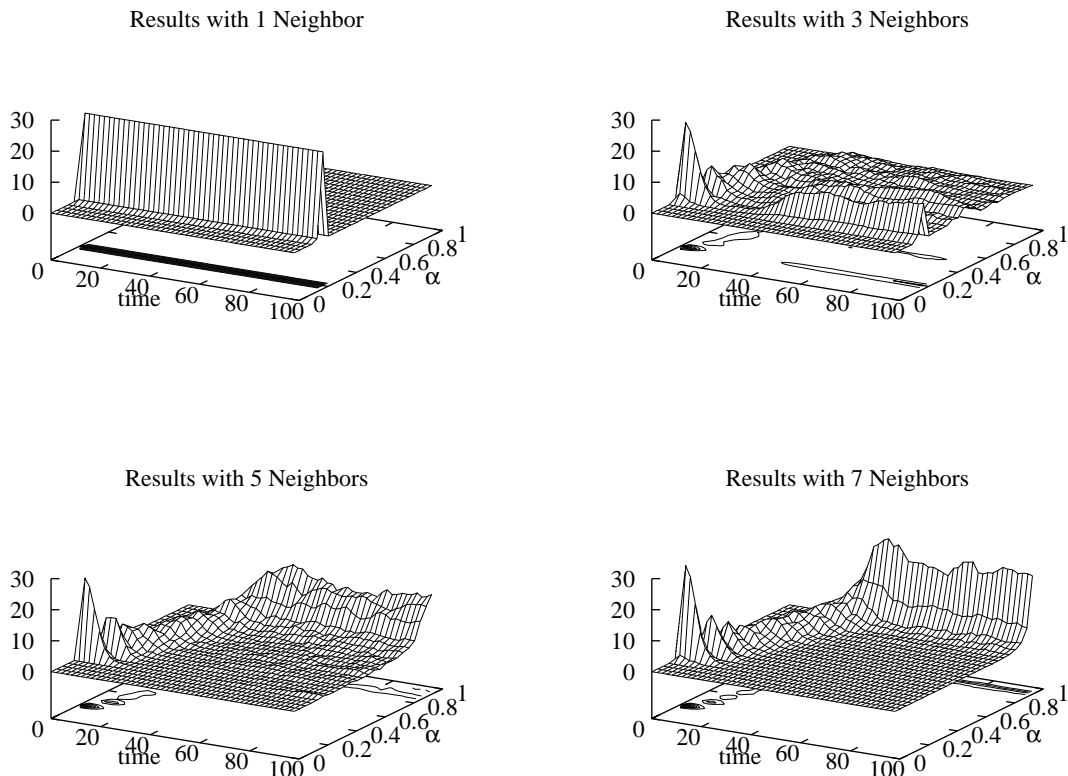


Figure 1: The z-axis on each figure represents the number of runs, out of 100, which had the particular α ratio at each particular time. $\alpha = 1$ means the run is at the global optimum. The optimum is reached more often in the cases with more communication.

will vary from domain to domain. We expect that our general approach can be applied to the design of any multiagent systems whose desired behavior is given by a global utility function but whose agents must act selfishly.

Our future work includes the study of how the system will behave under perturbations. For example, as the target moves it perturbs the current allocation and the global optimum might change. We also hope to characterize the local to global utility function correlation for different service allocation problems and the expected time to find the global optimum under various amounts of communication.

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