RUTCOR RESEARCH REPORT

AUCTIONS WITH ENDOGENOUSLY DETERMINED ALLOWABLE COMBINATIONS

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Abstract. Combinatorial auctions are desirable as they enable bidders to express the synergistic values of a group of assets and thus may lead to better allocations. Compared to other auctions, they keep bidders from being exposed to risks (of receiving only parts of combinations that would be valuable to them) or from being overly cautious (in order to minimize such risks). However, determining the optimal winning combination in a general combinatorial auction is computationally intractable, and this is sometimes given as a reason for not using combinatorial auctions. To insure computational tractability, a bid taker might try to limit the kinds of allowable combinations, but bidders may disagree on what combinations should be allowed, and this may make limiting the allowable combinations politically infeasible.
This paper proposes and tests successfully a new approach to managing the computational complexity of determining the winning combination. The main idea is to let bidders determine and prioritize the allowable combinations. Using endogenously determined combinations has two nice properties. First, by delegating the decision on what is biddable to the bidders who know what combinations are important to them, the bid taker is able to be (and appear) fair. Second, bidder prioritization of combinations assures that the economically most important combinations are included in determining the winning set of bids even when the bid taker is not able to use all the combinations submitted by bidders.

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1 Introduction

Game theoretic auction theory developed first with models of single, isolated auctions (McAfee and McMillan 1987). In the past few years, the academic literature has begun to pay attention to the design of auctions for selling large numbers of items with interrelated values. The FCC spectrum auctions, which involved the sale of thousands of licenses worth billion of dollars, helped motivate this attention (See Cramton 1995, 1997; Cramton and Schwartz 2000; McAfee and McMillan 1996; McMillan 1994), and the rise of auctions in electronic commerce is now also a motivator (Huhns and Vidal 2000).

One of the issues facing those designing auctions involving multiple items with interrelated values is deciding whether to allow bids on combinations of items, and if so, how to decide which combinations should be biddable. This presents a dilemma. Bidders may strongly prefer to bid on combinations when their values are for the combinations rather than for the individual items, and bid takers may prefer an auction in which bidders can make such bids. However, the mathematical problem of finding the revenue maximizing set of bids can be formulated as an integer programming problem that is NP-complete when bids on all possible combinations are allowed.

Rothkopf, Pekec and Harstad (1998) considered this problem. They suggested two approaches. When there is sufficient time, the auction's fairness can be maintained by letting all bidders as well as the bid taker have a chance to find solutions to the integer-programming problem and then selecting the solution that gives the highest total revenue. When there is not sufficient time or when the economically significant combinations are generally agreed upon, then combinations may be able to be limited so that the problem is guaranteed to be computationally manageable. They discussed and gave a variety of examples of the kinds of combinations that are computationally manageable and of the kinds that are not. They suggested that the bid taker might get the assistance of the bidders in identifying economically important combinations.

While bidders may sometimes agree on the economically important combinations, at times, there may be essential disagreements. Some bidders may have synergies between some kinds of items and others may have completely different kinds of synergies. In addition, bidders who have no synergies, may fear competition from those who do have them, and may oppose the use of bids on combinations or push for allowing bids on combinations that would be difficult to handle computationally in conjunction with their competitors' sincerely desired ones. In addition, for strategic reasons bidders may be reluctant to truthfully reveal the combinations of greatest interest to them. Bid takers who need to maintain the appearance of fairness may, thus, face difficulty in determining the most important combinations and a dilemma even if they do know which are economically significant. This paper discusses a possible solution to this dilemma—*an auction process in which the bidders themselves determine the allowable combinations.*

The auction process we propose is a one-time standard sealed bid auction in which bidders pay the amount of their accepted bids. Thus, it is an alternative to the simultaneous progressive multi-round auction used by the FCC. This has the advantage of limiting the opportunities for signaling and tacit collusion among bidders, but does not have the information sharing advantage (in the face of affiliated values) of the progressive format. As discussed below, however, it is possible to adapt our approach to simultaneous progressive auctions.

The basic idea of our approach is to let the bidders decide on and prioritize the combinations. Bidders will submit with their bids on individual items a priority list of combinations upon which they wish to bid along with bids on those combinations. The bid taker will first solve the revenue-maximization problem using the individual bids but no combination bids. This calculation involves no computational difficulties. The bid taker will then attempt to solve the revenue maximization problem using, in addition to the individual bids, each bidder's first priority combination bid.

The bid taker will have a time limit for this and subsequent calculations. If the time limit is reached before this calculation is completed, the bid taker will return to the last of the previously completed calculations and announce it as the results of the auction. If this calculation is completed before the time limit is reached, the bid taker will add each bidder's next priority bid to the bids to be considered and repeat the calculation. This process will continue until either the time limit is reached or all of the combination bids of all of the bidders have been considered. Figure 1 shows a flow diagram of this process.

This paper describes our approach in detail and reports and discusses a set of tests of it. After some tests with small problems, we put together a test problem roughly patterned on the FCC spectrum auctions involving the sale of 153 assets to three bidders with differing interrelated values for them.

First, we prepared our own set of bids for the bidders. The bidders had 15, 22, and 25 combinations on their priority lists. It turned out that computation was not a serious issue. The entire process, which involved the solution of 25 integer programming problems was completed using 14 seconds of CPU time on a 40 MHz Spark workstation using CPLEX 6.0. (Yes, 40 MHz, not 400MHz, and CPLEX 6.0, not 6.5 or 7.0, which are substantially faster.)

In our view, an important drawback of this test was that we made up the bids, and we knew every bidder's values when we did so. To remedy this deficiency, we enlisted the help of several economists who served as advisors to bidders in the FCC auctions. We asked each to bid for one of the bidders, knowing that bidder's values and only qualitative information about the situation of the other bidders.

In the following, we provide a detailed description of our algorithm and experimental results. Section 2 surveys the related work on combinatorial auctions, and discusses the current algorithms for winner determination in combinatorial auctions. Section 3 describes our algorithm in detail. Section 4 describes the test problem and experimental results. Section 5 concludes the paper. The appendix shows the letter sent to the test bidders.

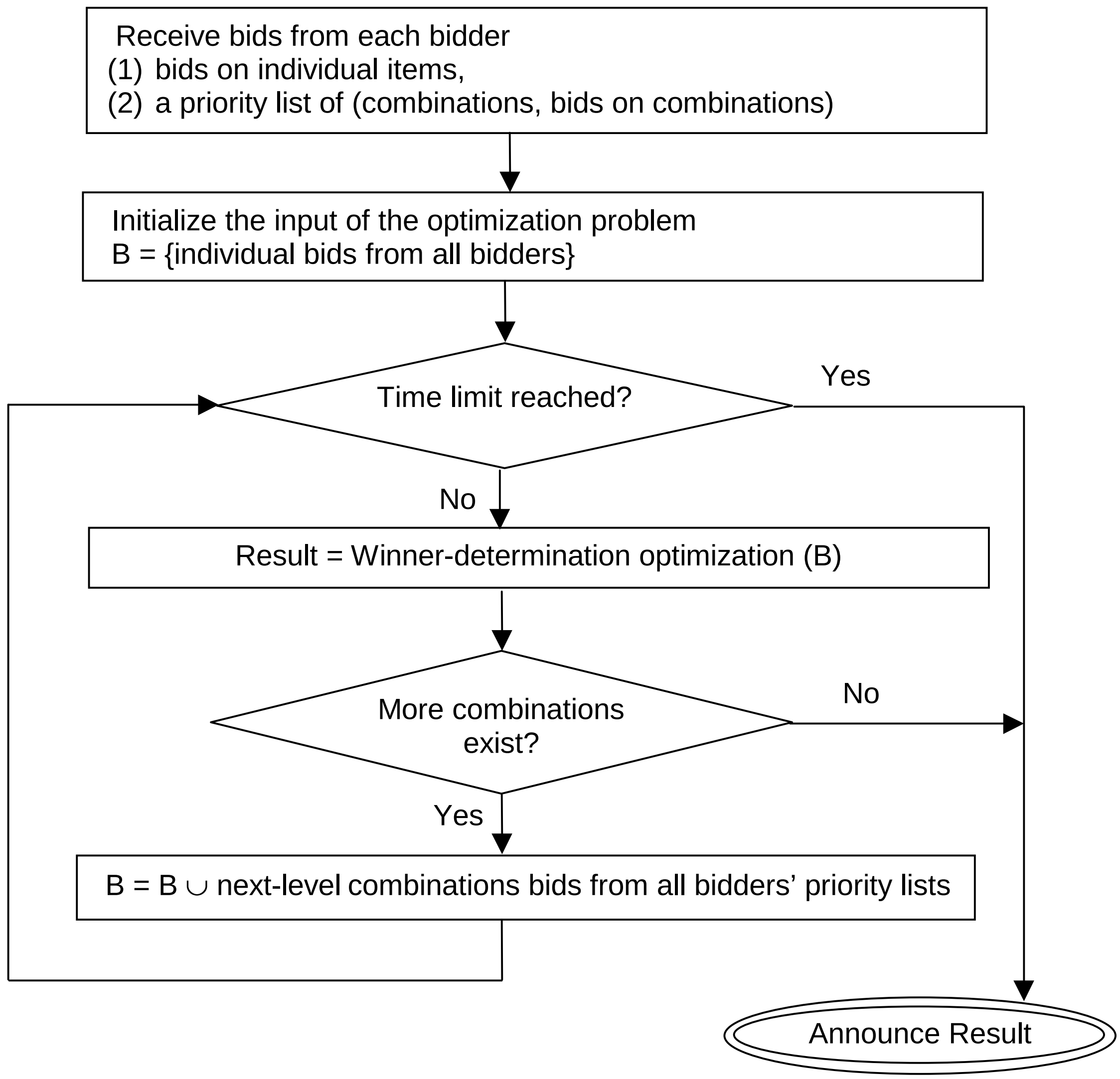

Figure 1: Flow chart of the auction process.

2 RELATED WORK

When heterogeneous, discrete assets with interrelated values are being sold in auctions, bidders often find that the value of one item depends on what other items they win. This may be due to several items complementing each other or their being partial substitutes for each other. Licenses to identical frequency spectrums in geographically adjacent areas in the FCC spectrum auctions are an example of the former (complementarities), while rights to different frequency spectrums in the same area are sometimes an example of the latter (substitutes). Other examples in which there are complementarities include airline landing slot auctions, hourly electricity markets, and railroad-segment auctions.

When the value of an asset to a bidder strongly depends on other assets she wins, it is desirable to allow bids on combinations of assets, as opposed to only on single items.

See the example in Figure 2 adopted from Wurman (1999). There are two assets, A and B, and two bidders, 1 and 2. Bidder 1 has no value for either asset alone, and has a value of $3 for the combination of goods A and B. Bidder 2 values either good at $2 each, but receiving both yields no increase in value. In this example, the most efficient allocation is to allocate both goods to bidder 1, but if bidder 1 does not know bidder 2's values, no separate auction that doesn't allow bids on the combination (A and B) will support this efficient outcome. That is, without bids on combinations, bidder 1 who has synergistic valuations is forced either to bear the risk of paying more for an individual item than it is worth, or to bid cautiously in a way that will not let her win combinations.

	A	B	A & B
Bidder 1	0	0	3
Bidder 2	2	2	2

Figure 2: Two bidders with two assets.

To date, several approaches have been proposed for the sale of assets with interrelated values. We divide these into roughly two groups. The first group does not use combinatorial auctions even when the synergies exist. These approaches may merely force bidders to speculate on risk without recourse. Alternatively, they may rely on an after market in which bidders who are stuck can seek further trades to improve their situation. Finally, this group includes the FCC style auction that sells multiple items in parallel in a multi-round auction in which bidders can observe competition developing, attempt to signal their interests and can withdraw at modest penalty from apparently failing aggregation attempts. (See Cramton 1995, 1997; Cramton and Schwartz 2000; McAfee and McMillan 1996; McMillan 1994.)

The second group of approaches attempts to use combinatorial auctions (that allow bidding on combinations of assets) in the presence of synergies among assets, since they allow the bidders to express their true preferences, and thus may lead to better allocations. For example, some (Rassenti *et al*. 1982; Banks *et al.* 1989; McCabe *et al.*1991) have proposed continuous-time auction mechanisms in which bidders may submit a bid on a combination if that bid increases the revenue over the bids it displaces.

Allowing bids on combinations, however, may create a computational burden. To select the winning bid combinations, the bid taker will attempt the optimization of some target function, usually the bid taker's revenue. In general, this optimization problem is NP complete, due to the exponential number of possible combinations (Rothkopf, Pekec and Harstad 1998). Therefore, many have focused on developing better algorithms or heuristics. (Fujishima 1999; Sandholm 1999; Parkes 2000; Kelly and Steinberg 2000). As noted by Andersson *et. al.* (2000), if the bid distribution is known to the bid taker, it is sometimes able to construct algorithms that capitalize on this knowledge.

Rothkopf, Pekec and Harstad (1998) discuss two different kinds of approaches on managing the complexity of winning bid selection in combinatorial auctions. The first approach allows a full set of combinatorial bids. It maintains fairness, by allowing bidders an opportunity to improve the sum of the bids whenever the bid taker's tentative bid selection is not provably optimal. In their second approach, they show several different kinds of limited sets of combinatorial bids that are computationally tractable.

For example, bids on any pair of items can be allocated as can bids for more than half of the items.

Our approach is related to their first group, as we allow a full set of combinatorial bids. However, we let the bidders assign priorities to various combinations and then use as many of these as is computationally possible (from the highest priority first) with the computational resources available.

3 THE FORMULATION AND THE ALGORITHM

In most auctions, the bid taker will attempt to allocate goods among the bidders in a way that the final allocation maximizes his revenue. In this section, we formulate the bid taker's revenue-maximizing optimization problem, and describe our proposed algorithm in detail.

3.1 FORMULATION OF THE OPTIMIZATION PROBLEM

Let G denote the set of all individual goods being auctioned. Let $C \subseteq G$ represent any possible combination of goods being auctioned (including the "combination" of a single item). Then, a bidder, i, can place a bid for combination C, $b_i(C)$. We let $b'(C)$ be the highest bid for combination C. If there is no bid for C, we set $b'(C) = 0$. Note that the $b'(C)$ are the only bids that can be included in the final revenue-maximizing outcome.

The bid taker's problem of finding a revenue-maximizing outcome is an integer-programming problem that can be formulated as follows:

$$\max \sum_C b'(C)\, x_C$$

subject to the constraints

$$\forall C : x_C \in \{0, 1\} \text{ and } \forall g \in G: \sum_{g\in C} x_C \leq 1.$$

The constraints require that in any outcome of an auction, the winning combinations must be disjoint since no single good can be sold more than once. More formally, let O denote the outcome of an auction, then

$$\forall\, o^1, o^2\ (\neq o^1) \text{ in } O,\ \ Co_1 \cap Co_2 = \emptyset.$$

Finding an optimal outcome is a weighted set packing problem, which is NP-complete (Rothkopf, Pekec and Harstad 1998). Without any restriction, the number of allowable combinations can be $(2^{|G|} - 1)$.

3.2 ALGORITHM

Figure 3 depicts the proposed algorithm. Suppose there are n goods, $G_1, \ldots, G_n$, and m bidders, $b_1, \ldots, b_m$. Each bidder i submits bids for single items j, $b_i(G_j)$. In addition, each bidder submits a priority list of combination and bid-price pairs. That is, bidder i's k-th priority list, *b[i].priority_list[k]*, is the tuple, $<C, b_i(C)>$.

At the first iteration, the allowable combinations are the bids on individual items with no other combination bids. Finding a revenue-maximizing allocation in this case is trivial. At the second iteration, the allowable combinations include the first combinations of every bidder's priority list (i.e., *B[i].priority_list[1]* for all *i*) in addition to the singleton bids. At the third iteration, the second combinations of the priority list (i.e., *B[i].priority_list[2]* for all *i*) are added to the previously allowable combinations. Of course, any bids that will never be part of an optimal outcome can be removed before solving the optimization problem. That is, only the highest bid for each allowable combination *C*, *b'(C)*, is used as input.

At each iteration, the bid taker solves the optimization problem using the current allowable combinations as input. We use a general-purpose integer programming software, CPLEX 6.0, for the optimization. If the time limit is reached during the optimization, the algorithm returns the previous optimization result and terminates. The revenue of the optimization outcome of the current iteration cannot be lower than that of the previous one. Hence, the current outcome determines the tentatively winning allocations.

This iterative process continues until the bid taker has exhausted all the combinations, or the pre-defined time limit is reached. This iteration may seem wasteful, but the cost of it is rather small. Intuitively, the reason for this is that the number of combinations (the critical factor in determining the computational time) increases with iterations, so the early iterations do not matter much compared to the later ones. Of course, if the bid taker were able to estimate reliably the time needed to solve each level of iteration, he could start from a higher level that can be finished within the time limit instead of from the first-level. In general, however, iteration is preferred when such reliable estimation is not available.

We have several initial comments on this process. First, shifting the burden of determining allowable combinations to bidders maintains the appearance of fairness of the bid taker. Of course, the worst-case complexity of the algorithm is still exponential, and therefore the algorithm may terminate without using up all the combinations submitted, but making the bidders prioritize their combinations makes them focus on the combinations with the greatest economic importance. Thus, even if only a small fraction of the combinations of interest to a bidder is used, most of the value obtainable from the use of combinations may be achieved.

Second, we note that the categorization of classes of problems made by computational complexity theory is based upon worst-case assumptions about the amount of computation involved for any member of the class. In practice, many problems in classes with bad worst cases are solved easily. To the extent that this is true here, our approach takes full advantage of it.

Third, there have been great advances in the last few years in our ability to solve the computational problems involved. In addition to the development of special purpose algorithms (Fujishima 1999, Sandholm 1999), general purpose integer programming is much faster now than it was a few years ago (Hobbs *et al.* forthcoming, section 2).

```
Input: Bids from each bidder, i
        (1) singleton bids, b_i(G_j)
        (2) a priority list of combination and bid-price pairs, b[i].priority_list[k]

flag = 0;                                          // Flag will be set to 1 when time limit is reached
B ← ∪_i b_i(G_j)  (for i = 1, …, m, and j = 1, … n) // Initialize input using singleton bids
Remove smaller bids from B                         // Only the highest bids remain from the input
O ← {};                                            // Initialize output

For (k=1; k<=MAX; k++) {                           // MAX is the maximum length of the priority list
    {flag, O_B} ← CPLEX_MIP(B);                    // Solve the optimization problem
    if (flag ==1) return O;                        // If time limit is reached, return
    if (revenue(O_B) > revenue(O))  O ← O_B;       // See whether the new allocation is better

    B ← B U ∪_i b[i].priority_list[k];             // Include the k-th priority list of all the bidders
    Remove smaller bids from B                     // Only the highest bids remain
}

return O;
```

Figure 3: The Algorithm of the auction process.

4 THE TESTS

To test the effectiveness of our algorithm, we created several test problems, all of which are modeled roughly on the economics of the FCC spectrum auction. We now describe one of them in detail, and examine the test results.

4.1 Problem Description

The test problem is based upon a 12 by 15 rectangular region in which three bidders are competing for a single asset in each region, except for 27 of the regions in which one of the bidders already owns an asset. Thus, there are 180 assets, 27 of which are pre-owned by bidders, leaving 153 to be sold.

Bidders have different "raw" values for different regions and somewhat different amounts of synergy for neighboring assets. Figure 3 depicts the raw values for bidder A, bidder B, and bidder C, respectively. Note that the raw values for the assets the bidders already own are represented in bold, and that the X's indicate the assets that are not for sale because the other bidders already own them.

The test problem is made to resemble the characteristics of FCC spectrum licenses. As in FCC spectrum licenses, there exist synergies among groups of assets. The synergistic addition to value of an asset to a bidder is a multiple of the values of "neighboring" assets that the bidder owns. Neighboring assets are assets that are horizontally or vertically (but not diagonally) adjacent to the asset. We use synergy multiples of 0.4, 0.3, and 0.2 for bidders A, B, and C, respectively. For example, bidder

A's value for asset (3,2) will be 13 + 0.4×21 (since she already owns (3,1)) + 0.4×7 (if she wins (2,2)) + 0.4×7 (if she wins (3,3)) + 0.4×7 (if she wins (4,2)). Thus, the test problem exhibits systematic economic synergies over the assets, as did the economics of the FCC spectrum auction.

4.2 Bids

First, we prepared our own set of bids for three bidders. We tried to make the bids as realistic as possible. First of all, when examining a bidder's strategy, we considered not just the *raw* value of assets, but groups of assets with the high *synergy*. We had the bidder try to assess its areas of competitive advantage and the size of that advantage but without assuming that bidders know each other's values very well. We also assumed that bidders were unsure how many combinations would get used by the bid taker's algorithm. Finally, we tried to pick bids in such a way that the bidder was balancing her uncertainty about the situation she faced against a desire not to drive up the price she paid by competing with herself.

All of the above provide a context in which bidders try to decide upon which combinations are important to them. In some cases, we made the bidder combine combinations that could have been bid separately in order to get higher priority for her combinations and to make these harder for the optimization to neglect. We prepared 15, 22, and 25 priority combinations for bidder A, B, and C, respectively. Then, we computed the value of each combination assuming that the bidder won nothing else. We used these values along with a sense of for which combinations the bidder had a competitive advantage to decide upon the bid prices.

In addition to preparing our own bids for the three bidders, we turned the test into a laboratory experiment. We requested several economists who had served as advisors to bidders in the FCC auctions for help in generating bids. Each of three bidders was given his own values and a rough idea of the situation faced by competitors (e.g., qualitative information about the competitors' raw values and about competitors' synergy multipliers). For reference, the letter sent to the bidders is shown in Appendix.

Due to space limitations, we do not list individual bids in this paper, but it is noteworthy to mention some strategies, both professed and inferred, adopted by the outside bidders. Many bidders tried to avoid including in their combinations bids on assets contiguous to the existing bases of the other bidders. One bidder decided not to make serious individual bids, in order to add pressure on the system to select one of his combination bids. With no better idea of how different the competitors' valuations are from his own, this might be a reasonable bidding strategy. A bidder A (who has relatively large synergies), on the other hand, launched a preemptive strategy aimed at winning large packages. One of the concerns among bidders was how not to bid against themselves. One bidder told us that he did not bid on overlapping combinations. Another bidder told us that he made only trivial individual bids on the individual assets while setting higher target profit margins on the smallest packages for the same reason.

A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
2	13	7	**X**	5	7	13	7	**X**	5	7	13	7	**X**	4	3.4
3	**21**	13	7	7	13	**21**	13	7	7	13	**21**	13	7	5	4
4	13	7	5	4	**X**	13	7	5	4	**X**	13	7	5	4	**X**
5	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
6	13	7	**X**	5	7	13	7	**X**	5	7	13	7	**X**	4	3.4
7	**21**	13	7	7	13	**21**	13	7	7	13	**21**	13	7	5	4
8	13	7	5	5	**X**	13	7	5	5	**X**	13	7	5	4	**X**
9	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
10	13	7	**X**	5	7	13	7	**X**	5	7	13	7	**X**	4	3.4
11	**21**	13	7	7	13	**21**	13	7	7	13	**21**	13	7	5	4
12	13	7	5	5	**X**	13	7	5	5	**X**	13	7	5	4	**X**

(a) Raw values for bidder A.

B	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
2	8	14	**22**	14	8	8	14	**22**	14	8	8	14	**22**	14	8
3	**X**	8	14	8	6	**X**	8	14	8	6	**X**	8	14	8	6
4	5	6	8	6	**X**	5	6	8	6	**X**	5	6	8	6	**X**
5	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
6	8	14	**22**	14	8	8	14	**22**	14	8	8	14	**22**	14	8
7	**X**	8	14	8	6	**X**	8	14	8	6	**X**	8	14	8	6
8	5	6	8	6	**X**	5	6	8	6	**X**	5	6	8	6	**X**
9	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
10	8	14	**22**	14	8	8	14	**22**	14	8	8	14	**22**	14	8
11	**X**	8	14	8	6	**X**	8	14	8	6	**X**	8	14	8	6
12	5	6	8	6	**X**	5	6	8	6	**X**	5	6	8	6	**X**

(b) Raw values for bidder B.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5.7	6	6.4	7	8	7	6.4	6.4	7	8	7	6.4	6.4	7	8
2	6	6.4	**X**	8	10	8	7	**X**	8	10	8	7	**X**	8	10
3	**X**	7	8	10	16	**X**	8	8	10	16	**X**	8	8	10	16
4	7	8	10	16	**24**	16	10	10	16	**24**	16	10	10	16	**24**
5	6.4	7	8	10	16	10	8	6.4	10	16	10	8	6.4	10	16
6	6	6.4	**X**	8	10	8	7	**X**	8	10	8	7	**X**	8	10
7	**X**	7	8	10	16	**X**	8	8	10	16	**X**	8	8	10	16
8	7	8	10	16	**24**	16	10	10	16	24	16	10	10	16	**24**
9	6.4	7	8	10	16	10	8	6.4	10	16	10	8	6.4	10	16
10	6	6.4	**X**	8	10	8	7	**X**	8	10	8	7	**X**	8	10
11	**X**	7	8	10	16	**X**	8	8	10	16	**X**	8	8	10	16
12	7	8	10	16	**24**	16	10	10	16	24	16	10	10	16	**24**

(c) Raw values for bidder C.

Figure 4: Raw values of the three bidders.

One possible strategy discussed informally (but not used by any of the bidders involved in the test) is to attempt to impede one's competition by bidding on additional combinations that would be computationally difficult (after the bidder uses up all the priorities she needs). However, it might be hard for a bidder to do this without putting herself at the risk of winning something she does not want, since branch and bound should not be troubled by a noncompetitive bid.

The number of combinations submitted by the bidders is shown in Figure 5. Including our own sets of bids, we have three bidder As, two bidder Bs, and two bidder Cs. Using all possible combinations of these results in twelve test cases.

Bidders	Number of Priority Combos
A1	16
A2	7
A3	5
B1	22
B2	42
C1	25
C2	16

Figure 5: Bidders and the number of combination bids submitted.

4.3 Experimental Results

Our primary concern is the speed of the algorithm. The entire process of selecting the winning combination, which involves the solution of up to 42 integer-programming problems, was always completed in less than 26.7 CPU seconds. This result was obtained on 40 MHz Sun Sparc 1000 machine with CPLEX 6.0, so we expect significantly shorter CPU times when using faster machines with CPLEX 6.5 or 7.0. Figure 6 gives the test results.

Test	Bidder A	Bidder B	Bidder C	Revenue	Needed level [1]	A's Value	B's Value	C's Value	Total Value	A's Profit	B's Profit	C's Profit	Total Profit	CPU time
1	A1	B1	C1	2697	1	1714	1345	1036	4095	601	364	430	1395	12.8
2	A1	B1	C2	2579	1	1802	1374	872	4048	620	382	466	1468	10.5
3	A1	B2	C1	2603	12	2018	1160	899	4077	696	396	380	1472	23.4
4	A1	B2	C2	2505	13	2018	1189	872	4079	696	411	466	1573	23
5	A2	B1	C1	2725	21	667	1535	1745	3947	428	395	400	1223	15.6
6	A2	B1	C2	2530	1	3166	198	216	3580	636	198	216	1050	12.1
7	A2	B2	C1	2698	12	2120	1102	873	4095	636	376	385	1397	26.7
8	A2	B2	C2	2530	1	3166	198	216	3580	636	198	216	1050	23
9	A3	B1	C1	2991	7	943	1345	1736	4024	267	364	400	1031	14.1
10	A3	B1	C2	2706	6	1532	1730	872	4134	353	609	466	1428	10.4
11	A3	B2	C1	3014	24	1664	953	1413	4030	345	311	359	1015	23.6
12	A3	B2	C2	2724	14	1824	1082	1106	4012	361	382	545	1288	22.9

Figure 6: Test results without time limit.

[1] The lowest priority level at which the maximum revenue is achieved.

The 'needed level' in Figure 6 represents the lowest priority level at which the winning allocation is found. We note that the ultimate winning allocation is often found at an early iteration. In four cases, it is found at the very first priority combination.

As a result, the overall efficiency (in terms of both revenue and value) does not decrease much even when the time limit is intentionally set very short. When limiting the calculation time to 10 seconds, we achieve the same results in 10 out of 12 cases, as shown in Figure 7. Even in the two cases (case 5 and case 11) in which the winning allocations are different, the revenues lost due to the time limit are relatively small. The extra revenue without time limit for case 5 is 28 (a 1.04% improvement), and for case 11 is 176 (a 6.20% improvement). In terms of allocation efficiency, the extra value without a time limit is even smaller. The "extra" value without a time limit for case 5 is -36 (-0.90%), and for case 11 is 10 (0.25%). We believe that making bidders prioritize their bids assures that all of the economically important combinations will be considered first.

Test	Bidder A	Bidder B	Bidder C	Revenue	Priority level	A's Value	B's Value	C's Value	Total Value	A's Profit	B's Profit	C's Profit	Total Profit	Max level[2]	Time limit[3]
1				Same as Figure 6											E
2															C
3															E
4															E
5	A2	B1	C1	2697	17	1000	1151	1832	3983	465	377	443	1285	17	E
6				Same as Figure 6											E
7															E
8															E
9															E
10															E
11	A3	B2	C1	2838	21	1824	1102	1094	4020	361	376	445	1182	21	E
12				Same as Figure 6											E

Figure 7: Test results with a 10-second time limit.

[2] The priority level used in optimization before the time limit.
[3] E means the time limit has reached before exhausting all the priority combinations. C means completed before the time limit.

Some observations can be made for each individual test. The revenue of the bid taker never decreases throughout iterations, as illustrated in Figure 8. This is to be expected as, at each iteration, the bid taker adds more combinations to the input without removing any. As a result, the revenue of the new outcome with the additional combinations must be at least as good as revenue of the previous outcome. We also note that although the total value does not necessarily increase throughout the iteration, at least one bidder's value does.

Round	Revenue	A's value	B's Value	C's Value	Total value	Total Profit
0	**2445.7**	**1112.8**	**1260.4**	**1497.3**	**3870.5**	**1424.8**
1	**2794.0**	**808.6**	**1260.4**	**1894.0**	**3963.0**	**1169.0**
2	**2873.8**	**838.6**	**1345.2**	**1813.2**	**3997.0**	**1123.2**
7	**2991.7**	**943.8**	**1345.2**	**1736.4**	**4025.4**	**1033.7**

Figure 8: The intermediate outcomes of test case 9.

Figure 9 depicts the final allocation of test case 9. As expected, the bidders receive the assets near to their pre-owned assets. Comparing it with the (seemingly) optimal allocation in Figure 10, we can see that bidder B and, especially, bidder A have allowed bidder C to win more than is optimal. As Figure 6 shows, bidder A3's strategy which is used in case 9 was generally less profitable than the other two A bidders' strategies.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	B	B	B	A	A	B	B	B	A	A	B	B	B	C
2	A	B	B	B	A	A	B	B	B	A	A	B	B	B	C
3	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
4	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C
5	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
6	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
7	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
8	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
10	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
11	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
12	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C

Figure 9: The winning combination of test case 9.

In a smaller yet similar test problem with only 17 assets (not shown in this paper), we were able to find the optimal allocation for the case where the bidders bid their true valuations on all the possible combinations, and compare the optimal allocation with the test results. However, we were not able to find the optimal allocation for the test problem with 153 assets. Instead, we tried to find a seemingly optimal allocation. We did this by making the bidders bid the true synergistic values of the combinations of assets nearest to the pre-owned asset, and incrementally adding more assets to the combination. We took care to make sure the combinations from bidders do not overlap each other. Figure 10 depicts this seemingly optimal allocation. Its total value is 4162.4.

Comparing it with the total values achieved by the experiments shown in Figure 6 suggests that there is some room for improvement. The total value achieved ranges from 87% to 99% of the seemingly optimal total value, with a median of 97%. However, comparing this with the other tests is not completely fair or realistic because the bidders are concerned about maximizing profits rather than achieving the maximum total value. Since with three bidders whose interests only partially conflict with each other, there is no reason to expect that an equilibrium set of bids would lead to the maximum efficiency.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	B	B	B	A	A	B	B	B	A	A	B	B	B	C
2	A	B	**B**	B	A	A	B	**B**	B	A	A	B	**B**	B	C
3		A	B	B	A	**A**	A	B	B	A	**A**	A	B	C	C
4	A	A	B	C	**C**	A	A	B	C	**C**	A	A	C	C	**C**
5	A	B	B	C	C	A	B	B	C	C	A	B	B	C	C
6	A	B	**B**	B	A	A	B	**B**	B	A	A	B	**B**	B	C
7	**A**	A	B	B	A	**A**	A	B	B	A	**A**	A	B	C	C
8	A	A	B	C	**C**	A	A	B	C	**C**	A	A	B	C	**C**
9	A	B	B	C	C	A	B	B	C	C	A	B	B	C	C
10	A	B	**B**	B	A	A	B	**B**	B	A	A	B	**B**	B	C
11	**A**	A	B	B	A	**A**	A	B	B	A	**A**	A	B	C	C
12	A	A	C	C	**C**	A	A	C	C	**C**	A	A	C	C	**C**

Figure 10: The seemingly-optimal allocation.

Note that a wide variety of allocations give values within a few percent of the seemingly optimal. For example, see Figure 11 which shows the allocation in test case 11. This case achieves 97% of the seemingly optimal value.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	A	B	B	C	A	A	B	B	C	A	A	B	B	C
2	A	A	**B**	B	C	A	A	**B**	B	C	A	A	**B**	B	C
3	**A**	A	B	C	C	**A**	A	B	C	C	**A**	A	B	C	C
4	A	A	B	C	**C**	A	A	B	C	**C**	A	A	B	C	**C**
5	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
6	A	A	**B**	C	C	A	A	**B**	C	C	A	A	**B**	C	C
7	**A**	A	B	C	C	**A**	A	B	C	C	**A**	A	B	C	C
8	A	A	B	C	**C**	A	A	B	C	**C**	A	A	B	C	**C**
9	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
10	A	A	**B**	C	C	A	A	**B**	C	C	A	A	**B**	C	C
11	**A**	A	B	C	C	**A**	A	B	C	C	**A**	A	B	C	C
12	A	A	B	C	**C**	A	A	B	C	**C**	A	A	B	C	**C**

Figure 11: The final allocation of test case 11.

5 DISCUSSION

Determining the winning allocation in combinatorial auctions is NP-complete, so it is not surprising that bid takers fear the worst-case scenario of not being able to determine the winners. A possible solution to the computational intractability is to limit the allowable combinations, but this may create a politically-charged problem of deciding which combinations should be allowed in the face of bidders with preferences for different combinations. Therefore, a bid taker who wants to appear fair[4] may avoid combinatorial auctions.

In comparison, our algorithm lets the bid taker be neutral; he does not have to make any decision on which combinations to allow. That decision is made by the bidders, each of whom knows her own preference. By delegating the decision on allowable combinations to bidders, the bid taker makes the auction fair to everyone.

Our auction algorithm, an anytime algorithm with general-purpose optimization software (CPLEX), performs very well. Under the realistic collection of bids (prepared by human participants), the computation time has not been an issue. Furthermore, the bidders were able to create their bids given realistic qualitative information about the other bidders. Thus, there is a significant class of situations for which computational complexity is not a bar to combinatorial auctions.

Bidding strategies under our auction mechanism need study. Such study will help to fine tune the bid distributions used in the experiments; realistic bid distributions should be based on not only the characteristics of the goods being sold but also some reasonable profit-seeking strategies. More importantly, a bidder who participated in our experiment

[4] Note that this may well be a matter of appearance only. The choice of no combinations can favor some bidders over others.

voiced concern about competing against himself when creating a priority list of combinations. Investigation is needed into the extent to which this is a problem and the extent to which bidders can avoid it.

The auction design we have considered is a one-time sealed bid auction. If it appears that the auction might lead to an inefficient allocation of items because bidders are too unsure of competitive values and aggressiveness to bid effectively (which is a potential problem in any sealed bid auction), then our approach can be adapted to an FCC style simultaneous progressive auction. This can be done by allowing bidders to submit priority lists of combinations in each round. The rounds can be tied together by making winning combinations in one round, automatically biddable (like singletons) for all bidders in the next.[5] Attention will need to be paid to defining appropriate activity rules. Note, however, that just as the procedure this paper studies carries the burdens of sealed bid auctions, a combinatorial multi-round auction will carry the burdens of simultaneous progressive auctions such as signaling by bidders to establish tacit collusion and stability in the face of explicit collusive agreements.

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[5] Technically, there is no guarantee that the combinations that won in one round will, with different bids, be computationally manageable in the next. However, given that the winning combinations were just a small fraction of those considered in the previous round and that the bids are likely to be "similar," it seems extremely unlikely that there will be a computational problem in practice.

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APPENDIX

The following is the letter sent to those assigned the role of bidder A. Similar letters were sent to those asked to bid as bidder B or C.

Thank you for agreeing to cooperate in our attempt to evaluate an approach to combinatorial auctions with endogenous determination of the allowable combinations. Enclosed is material describing the auction as seen by the bidder for whom you are going to bid. If you have any questions about this, please call or e-mail me. Please prepare your bids independently without consulting with other participants in this experiment, and mail them to me during the next couple of weeks. After we have received everyone's bids, we will evaluate the auction using your bids against all possible combinations of opponents and let you know the results.

THE GENERAL SITUATION

There are 180 "licenses" for a 12 by 15 grid of territories. There are three companies with a serious need for licenses. Each of the three already owns 9 licenses. The remaining 153 licenses are to be sold by auction. The auction will be by single round-sealed bid. Bidders will be able to submit bids on individual licenses. In addition, bidders can submit a priority list of combinations of licenses along with a bid on each combination on the list. The auctioneer will first select the revenue maximizing set of bids without using any combinations. Then she will use each bidder's first priority combination and attempt to solve the integer-programming problem of finding the revenue maximizing set of bids. If she succeeds within her time limit, she will then add each bidder's second priority combination and attempt to resolve the integer-programming problem before reaching her time limit. Again, if she succeeds, she will add each bidder's next priority combination and repeat the process. She will continue this until she has included all of the combinations on each bidder's priority list or she has run out of time. To test our approach, we plan to run this problem with a ten-second time limit on CPLEX 6.0 on a 50 MHz Spark 1000 computer with RISC technology. A test with bids I made up for this problem handled each bidder's first 17 combinations in the first 10 seconds, but, of course, the number of combinations covered will vary with the particular bids.

Each bidder has a private value for every possible combination of licenses it wins. The private value is comprised of two parts. One part is a stand-alone value for each license it wins. The other part is a synergy effect for the possession of neighboring licenses. Synergies occur only between licenses that are immediately adjacent in the grid vertically or horizontally (but not diagonally). Bidders have values for the licenses they already own and will keep these licenses and their values even if they win no licenses in the auction. However, these already owned licenses do benefit from synergies with adjacent licenses that the bidder wins. Each bidder is attempting to maximize the difference between the total value he gains from winning licenses he wins and the amount of his accepted bids.

ANSWERS TO SOME QUESTIONS:

a. Can a bidder include in a package bid licenses on which he did not bid singly?

Yes.

b. Can a bidder instruct that some subset of his set of bids already entered be withdrawn in the process of going to his next prioritized package bid?

No, but if a package conflicts with some other bids of his, both will not be honored.

c. Can a bidder give more than one package bid the same priority?

No, but he can combine them into one package.

d. If a bidder wants to have a package bid considered higher priority than one of his bids on a singleton, can he make that singleton bid a trivial package, and give it lower priority?

No. However, if a bidder wants a bid on a singleton to be considered only if bids on certain packages are considered, he has the option of not bidding on the singleton as a singleton but of including a bid on that singleton as one of his combinations.

e. Can a bidder have an empty priority level, and nonempty lower priority levels?

Yes; he can achieve this by bidding submitting a bid for a combination or singleton upon which he has already bid as the combination in the priority level he wants to keep empty.

f. Can a bidder's package submission be contingent? For example, could a bidder’s package bid for licenses A, B, C, D, and E depend upon his not currently being the winning bidder on all five (or on four of the five)?

No; no bid is contingent in any way.

THE PARTICULAR SITUATION FACED BY BIDDER A

You are bidder A. You own licenses at the following locations on the grid: (3,1), (3,6), (3,11), (7,1), (7,6), (7,11), (11,1), (11,6), and (11,11). The attached table gives your stand-alone value for each license for sale (and, in bold face, for the nine you already own). In addition to the stand-alone values of the licenses you win, you will also get a synergy effect on (vertically or horizontally) adjacent licenses of 40%. For example, suppose you win licenses at (5,1) and (5,2). The license at (5,1) would be worth 9, its stand-alone value of 7 plus 2 which is 40% of the value of (5,2). The license at (5,2) would be worth 7.8, its value of 5 plus 40% of the value of (5,1). If you also won the license at (4,1), it would be worth 24.2, 13 plus 40% of the value of (3,1) which you already own and of (5,1). In addition, winning (4,1) would increase the value of each of (3,1) and (5,1) by 5.2, which is 40% of its value of 13.

You have some imprecise knowledge about the values of your competitors. You know that bidder B has licenses at (2,3), (2,8), (2,13), (6,3), (6,8), (6,13), (10,3), (10,8), and (10,13) and that bidder C has licenses at (4,5), (4,10), (4,15), (8,5), (8,10), (8,15), (12,5), (12,10), and (12,15). You know that, like your own values, competitors’ values are likely to be higher near the licenses they already own. You have reason to think that both of the other bidders may have slightly higher stand-alone values than you do, especially bidder C, but that they have somewhat smaller synergies than you do, especially bidder C. You have reason to believe that your competitors have a similar degree of knowledge about your situation and about each other’s.