AN EQUAL EXCESS MODEL OF COALITION FORMATION[1]

by Samuel S. Komorita

University of Illinois

Current theories of coalition formation in decision processes of systems at the level of the group lack generality in that some theories do not predict which coalitions are likely to form while others make predictions in only a narrow range of conditions. A model of coalition formation is proposed which predicts coalition formation in a variety of situations. Comparisons between predicted and observed results of studies using different coalition paradigms show considerable empirical support for the model. Despite some ambiguity in specifying the effects of some situational variables, the model provides a framework with which to study the processes of coalition formation and to examine the effects of such situational factors.

KEY WORDS: group systems, decision processes, coalition formation, n-person game, bargaining.

SINCE THE pioneering work of Von Neumann and Morgenstern (1947), a variety of solution concepts for n-person cooperative games have been proposed (cf. Luce & Raiffa, 1957; Rapoport, 1970). However, these solution concepts do not yield predictions about the likelihood of various coalitions, and despite these theoretical developments, very little progress has been made to develop a theory that predicts which coalitions are likely to form, as well as the payoffs to the coalition members. Although several descriptive theories that predict both of these response measures have been proposed (Gamson, 1961a; 1964; Komorita & Chertkoff, 1973; Komorita, 1974), all of these theories are restricted to situations in which the value (reward, prize) associated with all "winning" coalitions is constant (a simple game). A more serious problem is that such descriptive theories require the assignment of resources (weights, votes, etc.) to each of the players, and without such resource assignment, predictions cannot be derived. The purpose of this paper is to describe and evaluate a theory of coalition formation that predicts both coalition formation and reward division in situations in which the prize is either constant or variable, and where resources are not assigned to the players.

THE EQUAL EXCESS MODEL

The basic principle underlying the equal excess model is that an individual's "bargaining strength" in negotiations is based on the alternative coalitions the person can form. This conceptualization of bargaining strength (bargaining power) is based on Thibaut and Kelley's (1959) concept of "comparison level for alternatives," and is also the basis of the bargaining theory (Komorita & Chertkoff, 1973) of coalition formation. A second basic principle underlying the theory is that members of a potential coalition are most likely to agree on a division based on sharing equally the excess of what can be gained by the coalition, relative to the total outcomes if each chose his best alternative. This idea of equal shares of the excess is a derivation of Nash's solution (Nash, 1950, 1953) to the general bargaining problem, and is consistent with the predictions of Aumann and Maschler's (1964) bargaining set.

As with almost all theories of coalition formation, we shall assume that individuals are motivated to maximize expected payoffs. However, unlike other theories, we shall also assume that maximizing expected reward is not the only motivational basis of coalition behavior, and shall assume that

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social psychological motives also play an important role. In addition, we shall assume that the value of each coalition represents utilities and that utilities are transferable and conserved (cf. Luce & Raiffa, 1957; Rapoport, 1970). To simplify the presentation, we shall first restrict ourselves to the case in which the value of the grand coalition (the coalition of all players) is zero, and illustrate the predictions of the model with the following three-person game described by Riker (1971):

$$v(A) = v(B) = v(C) = 0;\ v(ABC) = 0;$$
$$v(AB) = 6;\ v(AC) = 5;\ v(BC) = 4;$$

where A, B, and C denote the three players, and $v(\ )$ denotes the value of each possible coaliton.

In the initial stage of coalition formation persons must decide which of the possible coalitions they wish to form, and given this choice, they must decide on their initial demands in the negotiations. Assumptions (1) and (2) specify the preferences of the individuals and the nature of their expectations during the various stages of negotiations.

Assumption 1. In the prenegotiation phase of coalition formation, individuals will prefer and attempt to form the coalition that maximizes initial expectation, given by:

$$E^0_{iS} = v(S)/s \tag{1}$$

where E^0_{iS} denotes the initial expectation of individual i in coalition S; $v(S)$ denotes the value of coalition S; and s denotes the number of persons in coalition S.

For Riker's three-person game described earlier, the initial expectations in each of the two-person coalitions are: 3 each for A and B in the AB coalition; 2½ each for A and C in the AC coalition; and 2 each for B and C in the BC coalition. Since the AB coalition maximizes initial expectation for both A and B, Assumption (1) implies that A and B are most likely to initiate negotiations. In negotiating the division of rewards, it is assumed that the players will base their demands on their expectations in alternative coalitions: The better the quality of their alternatives, relative to the alternatives of other players, the greater their bargaining strength (Thibaut & Kelley, 1959; Komorita, 1977). Since there will be a discrepancy between the expectations (demands) of the players, one or more players must make concessions to reach an agreement. Assumption (2) specifies the nature of concessions that are likely to be made during various stages (rounds) of negotiations.

Assumption 2. The expectations of the players will change over successive rounds of negotiations. Based on such changes in expectations, if a coalition forms after a given round of offers, the most likely coalition is one in which such expectations are jointly maximized for all players in the coalition, and the most likely reward division is given by the equal excess norm, as follows:

$$E^r_{iS} = \max_{S\neq T} E^{r-1}_{iT} + \left(v(S) - \sum \max E^{r-1}_{jT}\right)/s \tag{2}$$

where E^r_{iS} denotes the expectation of individual i in coalition S on round r; $\max E^{r-1}_{iT}$ denotes individual i's maximum expectation in alternative coalitions on round $r - 1$; and the summation is over the members of coalition S. A round will be defined as a sequential process in which each person makes an offer or counteroffer, and each person accepts or rejects offers which were received.

The numerator term of Eq. (2) denotes the difference between the value of coalition S and the sum of the maximum alternative expectations of the players in coalition S. It can be seen that if $v(S)$ exceeds the sum of the players' expectations, Assumption (2) implies that they will agree to divide the excess equally. On the first round of negotiations, if players A and B decide to negotiate with each other, player A's maximum expectation is 2.5 in the AC coalition, while B's maximum expectation is 2.0 in the BC coalition. Substituting these values of $\max E^0_{iT}$ in Eq. (2), we have:

$$E^1_{A(AB)} = 2.5 + [6.0 - (2.5 + 2.0)]/2 = 3.25,$$
$$E^1_{B(AB)} = 2.0 + [6.0 - (2.5 + 2.0)]/2 = 2.75.$$

Hence, if the AB coalition should form on the first interchange of offers, the model predicts that the most likely reward division is 3.25–2.75 for players A and B, respectively.

It is implicitly assumed that there will be individual differences in the competitiveness of the players, e.g., some persons assigned to position A may accept a share of 3.25, but others may demand a larger share. Hence, for persons with a high level of aspiration, additional rounds (offer–counteroffer sequences) may be required to reach an agreement.

For subsequent rounds of offers, Table 1 shows the predictions of the model based on iterations of Eq. (2). It can be seen that the expectations of player A increase over successive rounds of negotiations, in both the AB and AC coalitions, while the expectations of player C decrease in both the AC and BC coalitions. The expectation of player B, on the other hand, increases in the BC coalition, but decreases in the AB coalition. At the asymptote, note that the predicted shares of the players equal their maximum expectations in alternative coalitions. This implies that once an agreement is reached at the asymptotic predicted value, the players will not be tempted to defect from the coalition.

Since each player is assumed to maximize expectation, Table 1 implies that if a coalition forms in the early rounds of negotiations, the AB coalition is most likely to occur, and the BC coalition is least likely to occur. However, as the number of rounds required to reach agreement increases, the likelihood of the AB coalition should decrease, and the likelihood of the BC coalition should increase. Theoretically, if an indefinite number of rounds are required (the asymptotic expected values), the three coalitions should be equally likely. This predicted relationship between bargaining duration (rounds) and coalition outcomes is consistent with the results obtained by Friend, Laing, and Morrison (1977) on their computer simulation studies of "sequential games of status." They found large differences in the frequencies of various coalitions when bargaining persistence (trial duration) was low, but when bargaining persistence was high, coalition frequencies were closer to equal likelihood. Moreover, similar effects of bargaining persistence on coalition outcomes were found by Vinacke (1962).

The predicted changes in expectations over rounds are also consistent with the predictions of the bargaining theory of coalition formation (Komorita & Chertkoff, 1973), and have been supported in a study by Komorita and Moore (1976). The basis of these changes in expectations is that both players B and C are expected to send offers to player A in the initial encounter. These offers are expected to increase player A's level of aspiration and to increase his demands on subsequent rounds. Correspondingly, the increasing demands of player A are expected to lower the levels of aspiration of players B and C in the AB and AC coalitions, respectively, and to decrease their demands on subsequent rounds.

ASSUMPTIONS ABOUT THE BARGAINING PROCESS

Assumptions (1) and (2) specify the preferences of the players in the initial stage of coalition formation, and the expectations of the players over successive rounds of negotiations. It was also implied that individual differences in level of aspiration (competitiveness) can be expected. If all players are highly competitive, many rounds of negotiations are likely to be required before an agreement is reached, and the asymptotic expectations are likely to yield the best estimate of payoffs. However, if the players are not competitively motivated, few rounds may be required, and the expectations on the early rounds are likely to

TABLE 1
PREDICTIONS OF MODEL FOR RIKER'S (1971) THREE-PERSON GAME.

Coalition	Expectations over rounds						
	0	1	2	...	5	...	Asymptote
AB	3.0–3.0	3.25–2.75	3.37–2.62	...	3.49–2.51	...	3.50–2.50
AC	2.5–2.5	3.00–2.00	3.25–1.75	...	3.48–1.52	...	3.50–1.50
BC	2.0–2.0	2.25–1.75	2.38–1.62	...	2.48–1.52	...	2.50–1.50

yield the best estimate of payoffs. Thus, there is a certain degree of indeterminancy built into the model regarding the round on which an agreement is likely to be reached.

Although a round was defined as a process in which each person makes an offer or counteroffer and each person accepts or rejects any offers received, the nature of such a sequence would depend upon the nature of the experimental situation. For example, a round would be quite different if reciprocity of choices is required prior to negotiations between the players, as in Gamson's (1961b) "convention paradigm," as contrasted with a round where all players are allowed to communicate in a face-to-face situation. This assumption about the effects of situational factors is consistent with Riker's (1971) distinction between the informal (bargaining) rules of a game and the formal rules (characteristic function) of the game. More importantly, this assumption follows directly from the findings of Friend, Laing, and Morrison, who found that bargaining persistence and duration had important effects on coalition outcomes. They hypothesize that bargaining persistence "increases with extrinsic motivation (e.g., magnitude of game reward), intrinsic motivation (e.g., competitiveness of players), and ease of communication (e.g., face-to-face bargaining versus note passing), and decreases with time pressure to reach agreement or other time-related costs" (1977, p. 296). Accordingly, it will be assumed that the following situational variables determine whether an agreement is reached in the early or late rounds of negotiations:

(1) *Competitiveness of the players.* To the extent that all players are highly motivated to maximize reward, an agreement is likely in the late stages of negotiations.

(2) *Size of stakes.* If the players are negotiating for points or for very small monetary stakes (relative to the value of time required to reach agreement), an early agreement is likely. However, if the stakes are relatively large, an agreement in the later stages is more likely.

(3) *Familiarity and experience with coalition games.* It is assumed that sophisticated, experienced bargainers are likely to converge to the asymptotic predicted values at a faster rate than inexperienced bargainers. It is plausible that experienced players are more likely to recognize when their demands are "unreasonable" and when concessions are necessary to disrupt coalitions that exclude them.

(4) *Restrictions on communication and the amount of information.* Assuming players seek information regarding the preferences and expectations of the other players, any procedure that restricts communication among the players is likely to inhibit changes in expectations predicted by the model. For example, if a tentative agreement is reached by members of a potential coalition, and if this information is not available to the excluded players, they will not know that concessions are necessary to disrupt the tentative coalition. Hence, if they fail to make tempting offers to one or more members of the tentative coalition, a permanent agreement may be reached by the coalition members on an early round.

In accordance with the above hypotheses regarding the situational variables that are likely to affect the round on which an agreement is reached, we shall adopt the following criteria ("working hypotheses") to evaluate the predictions of the proposed model: (1) If the results of an experiment are based on paid volunteers; who played the game for relatively large stakes, in a repeated measures design, with few restrictions on communication and information; the asymptotic values will be used; (2) if subjects are required to participate (recruited from a "subject pool"), played for relatively small stakes, and some restrictions on information and communication are imposed, then the round 1 predictions of the model will be used.

The validity of threats

In negotiating the division of rewards, Assumption (2) implies that an individual's "bargaining strength" is based on the quality of his alternatives, and to justify his share of the prize, each person is expected to appeal to his maximum expectation in alternative coalitions. Such threats to defect to an alternative coalition will be defined as *valid* (credible) only if the alternative coalition does not include other members of coalition S. Otherwise, when such threats are made the other members can counter the threat by refusing to join the alternative coalition. This definition of valid alternative is comparable to Aumann and Maschler's (1964) concepts of "justified objection and counterobjection," in their bargaining set theory of coalition formation. Because of this commonality, for many types of games, the two models make similar predictions regarding the reward division among the coalition members.

To illustrate this definition of valid threats, consider the following four-person game:

$$v(A) = v(B) = v(C) = v(D) = 0;\ v(BC) = v(BD) = v(CD) = 0;$$
$$v(AB) = v(AC) = v(AD) = 100;$$
$$v(ABC) = v(ABD) = v(ACD) = v(BCD) = v(ABCD) = 100.$$

This game is called a *simple game*, a class of games in which there are two subsets of coalitions: "winning" and "losing" coalitions. The value of all "winning" coalitions is constant (100 in this example) and the value of all "losing" coalitions is also constant (zero in this example).

Based on Eq. (1) and (2), Table 2 shows the expectations of the players for all of the "winning" coalitions, except the grand coalition ($ABCD$); the expectations for the grand coalition is not included because they are uniformly less than those for the other coalitions. It can be seen that the two-person coalitions (AB, AC, AD) mutually maximize expectations for all players, and the model predicts that these coalitions are most likely to occur. For the purpose of illustration, however, let us suppose that players A, B, and C have temporarily agreed to form the ABC coalition and are negotiating the division of rewards. According to Assumption (2), in order to maximize his share of the prize, person A will threaten to defect to the AD coalition, the coalition with the maximum expected reward (his best alternative). This alternative excludes both B and C, and consequently, it is a valid alternative. Now what alternatives do B and C have? Player B might threaten to form the AB coalition, but player A is one of the members he is threatening. Since A can refuse to join him, this is not a valid alternative (credible threat). Although B and C will not have a valid alternative (according to this definition), it is reasonable to assume that they would use the threat of the BCD coalition against A. Hence, it will be assumed that if there is a coalition for a subset of players—each of whom do not have a valid alternative as individual players—such a coalition will be used as a threat against the other members of the potential coalition, and will be defined as a *valid alternative for the subset.* This idea of valid alternatives for a subset is comparable to Horowitz's (1973) extension of the bargaining set, to allow objections and counterobjections for subsets of the players. The main implication of threat credibility is that max E_{iT}^{r-1} in Eq. (2) is restricted to the valid alternatives of each player or subset of players.

Reciprocity, trust, and coalition formation

In a coalition situation individuals are faced with a conflict between maximizing their respective share of the prize and maximizing the probability of being included in the winning coalition. However, these two factors are inversely related: The larger an individual's demands, the smaller the chance that they will be accepted. For example, in the four-person simple game, if player A persistently demanded a 70–30 split in the AB, AC, and AD coalitions, he would encourage the three "weaker" players to form the BCD coalition, the "weak-union," for an equal split of 33 each. But such unreasonable demands by player A may have very important consequences in

TABLE 2
PREDICTIONS OF MODEL FOR FOUR-PERSON SIMPLE GAME.

Coalitions	Expectations over rounds						
	0	1	2	...	5	...	Asymptote
AB, AC, AD	50–50	58–42	63–27	...	66–34	...	67–33
BCD	33–33–33	33–33–33	33–33–33	...	33–33–33	...	33–33–33
ABC, ABD, ACD	33–33–33	44–28–28	50–25–25	...	56–22–22	...	56–22–22

subsequent encounters. Once the three weaker players begin to initiate offers to each other, a "common affective bond" may develop among them. If the weaker players reciprocate offers for a period of time, they will have made a tentative commitment to form the weak-union, and it may become exceedingly difficult to defect from this commitment. Based on these considerations, the following assumption is made about the bargaining process.

Assumption 3. When the members of a potential coalition reciprocate offers with each other, a social norm (a commitment) is likely to develop to form this coalition. The larger the number of persons who make such reciprocal offers, and the more often they reciprocate offers, the more difficult it will become to defect from the commitment.

The most important implication of Assumption (3) is that motives other than maximizing reward are assumed to affect coalition likelihood. For example, if the members of the coalition predicted by Assumptions (1) and (2) begin to negotiate with each other, one or more members of the coalition may threaten to form alternative coalitions. If attempts are indeed made to form such alternative coalitions, feelings of trust and mutual accommodation will be reduced considerably, and the other members are likely to retaliate. Obviously, such an escalation of the conflict is not likely to facilitate a mutually agreeable solution.

According to Assumption (3), therefore, the more often members of a potential coalition reciprocate offers with each other, the more likely it is that it will actually form in the future; conversely, the more often one or more members do not reciprocate offers, the more likely it is that attempts will be made to form alternative coalitions. Assumption (3), though somewhat tautological, is intuitively plausible: If a player makes repeated offers to form a given coalition, and if such overtures are not reciprocated, he is likely to lose all hope of forming the coalition and his expectation in the coalition should decrease markedly.

Another important implication of Assumption (3) is that it leads to the prediction that certain coalitions may be more likely than others—despite the fact that expectations may be equal for the various coalitions. For example, in Riker's three-person game, the model predicts that expectations should converge asymptotically to a solution of 3.5–2.5 for the *AB* coalition, 3.5–1.5 for the *AC* coalition, and 2.5–1.5 for the *BC* coalition (Table 1). The asymptotic expectations of the three players are therefore equal for their respective alternatives. However, Assumption (3) implies that if players *B* and *C* have not been reciprocating offers to each other (implied by predictions on previous rounds), then at the asymptotic level the *AB* and *AC* coalitions will be more likely to occur than the *BC* coalition.

With regard to the empirical support for Assumption (3), Gullahorn and Gullahorn (1963, 1964) simulated social interaction in the triad, and found that simulated triads tended to develop into a friendly pair and an isolate. This outcome could be traced to a process in which a pair of individuals mutually reinforced each other early in the interaction sequence, and gradually led to the exclusion of the third person. Hence, Coleman (1964, 1965) has argued that coalition formation in the triad can be explained on the basis of early mutual reinforcement, and one need not invoke more complicated assumptions to explain such outcomes. There is also indirect support for Assumption (3) from the results of studies by Esser and Komorita (1975) and Komorita and Esser (1975), who found that in a dyadic bargaining situation a preprogrammed strategy of reciprocating subjects' concessions yielded a higher proportion of agreements than when concessions were not reciprocated.

THE CASE OF SUPERADDITIVE GAMES

When the value of any coalition, including the grand coalition, is greater than or equal to the sum of the values of any disjoint subsets of the coalition, it is called a *superadditive* game. Our previous examples were nonsuperadditive games, and in such cases there is little incentive to form large coalitions. In Riker's three-person game, suppose the value of the grand coalition, hereafter denoted $v(G)$, is equal to 6.0 instead of zero. According to Eq. (1), the initial expectation (E^0_{iG}) will be 2.0 for each player. Since this expectation is less than the expectation in the AB and AC coalitions (see Table 1), the model predicts that none of the players will attempt to form G. Since iterations of Eq. (2) yield expectations in G that are uniformy less than those in the two-person coalitions, when $v(G) = 6$, the model predicts that the grand coalition is not likely to occur.

Suppose, however, that $v(G)$ is larger than 6.0. Table 3 shows the expectations of the players when $v(G)$ is equal to 6.0, 7.5, and 9.0. When $v(G) = 7.50$, the initial expectation in G is 2.50 for each player. In comparison with the values for the two-person coalitions (Table 1), both players A and B will initially prefer the AB coalition, but player C will be indifferent between AC and G. On round 1, however, player C will prefer G because it maximizes expectation, but on what basis can he justify his expectation of 2.17 in G? Note that in negotiations among all players in a game, none of them will have valid alternative coalitions as individual players, but all will have valid alternatives as subsets of players. This means that each pair of players can threaten the third player, and such threats to form the two-person coalitions should be highly credible. Accordingly, in the case where all players agree to form G and decide to negotiate the payoff division, it will be assumed that each player will appeal to, and base his demands on, his maximum expectation in a subset of the grand coalition. Thus, when $v(G) = 7.5$, the model implies that players A and B will use the threat of the AB coalition against player C to lower his expectations and his demands. Assuming player C lowers his demands as predicted by the model, at about the third round of negotiations player B will be relatively indifferent between AB and G, and by the fifth round all players will be relatively indifferent between the grand coalition and their expectations in the two-person coalitions. Hence, the model implies that if player C lowers his demands in the early rounds, he will induce player B to reciprocate offers to form the grand coalition, and if players B and C induce player A to reciprocate offers to form G, according to Assumption (3), the grand coalition should be likely to occur. With regard to the predicted payoffs when $v(G) = 7.5$, note that the asymptotic expectations of the players are in equilibrium (the unique core point) so that none of them will be tempted to defect.

Finally, when $v(G) = 9$, Table 3 shows that player C will prefer G at the outset, while players A and B will be indifferent between AB and G. By round 2, however, all players will prefer G, and the grand coalition is predicted to be highly likely. These examples suggest that the likelihood of the grand coalition increases monotonically with its value; moreover, as $v(G)$ increases in value, an agreement to form G becomes more likely in the early stages of negotiations.

EVALUATION OF THE MODEL

To evaluate the validity of the proposed model, we shall first consider some data reported by Riker (1971) for the three-person game which we used to illustrate the predictions of the model (Table 1). Riker replicated this game in seven experiments (total of 149 subjects), and although the procedure and subject populations were not entirely comparable, we shall pool his results over the seven experiments. In the seven experiments there were 205 replications of this game, and Table 4 shows the

TABLE 3
PREDICTIONS OF THE MODEL FOR RIKER'S THREE-PERSON GAME WHEN THE VALUE OF THE GRAND COALITION (ABC) EQUALS 6.0,7.5, AND 9.0.

$v(ABC)$	Expectations over rounds						
	0	1	2	...	5	–	Asymptote
6.0	2.00–2.00–2.00	2.17–2.17–1.67	2.58–2.08–1.33	...	2.95–2.01–1.04	...	3.00–2.00–1.00
7.5	2.50–2.50–2.50	2.67–2.67–2.17	3.08–2.58–1.83	...	3.45–2.51–1.54	...	3.50–2.50–1.50
9.0	3.00–3.00–3.00	3.17–3.17–2.67	3.58–3.08–2.33	...	3.95–3.01–2.04	...	4.00–3.00–2.00

TABLE 4
MEAN PROPORTION OF OCCURRENCE OF COALITIONS (p) AND MEAN REWARD DIVISION* IN RIKER'S (1971) THREE-PERSON GAME.

Coalitions		
AB p Division	AC p Division	BC p Division
.46 (3.44–2.56)	.30 (3.49–1.51)	.24 (2.49–1.51)

* Shown in parentheses.

frequencies of occurrence of the three coalitions and the mean payoffs in these coalitions. Since the model predicts that the AB coalition should be most frequent (the AB coalition was most frequent in six of the seven experiments), the frequency data provide partial support for the model.

With regard to the predicted payoffs, the asymptotic values for this game (Table 1) coincide with the predictions of Aumann and Maschler's (1964) bargaining set. In six of the seven experiments, subjects were given special instructions in the game, designed to make them sophisticated bargainers. Moreover, since relatively large stakes were involved ($4, $5, and $6 per pair), based on our assumption about situational factors and rounds to reach agreement, we shall use the asymptotic values as an estimate of the reward division. Table 4 shows that the asymptotic values yield a very good estimate of the obtained values (except for the slightly low mean payoff for player A in the AB coalition).

In describing the results for one of the seven experiments, Riker (1967) contrasts his results with the results obtained by Maschler (1965) and by Lieberman (1962). Both Riker's and Maschler's results supported the predictions of the bargaining set (as well as the proposed model). Lieberman, however, found a large proportion of cases (182 out of 320) in which the payoffs were an equal split, thus contradicting the predictions of the bargaining set. Accordingly, Riker (1967) argued that:

> The differences among these experimental outcomes may be largely accounted for by institutional factors built into the experimental design. Maschler's subjects bargained face-to-face lengthily. As might be expected, therefore, his subjects came close to the bargaining set, just as mine did. Lieberman's subjects, on the other hand, communicated by turning cards and for only a few moments. Since they were rushed for time, they chose what is probably the easiest method of generating trust quickly, namely, equal division (p. 65).

This explanation is, of course, completely consistent with our assumption about situational factors and rounds to reach agreement: The greater the time pressure to reach agreement, the more likely an agreement on the early rounds of negotiations, and the more likely an equal division of the payoffs (round 0 estimate in Table 1). It should be noted, however, that Lieberman (1971), in a reply to Riker's explanation, appropriately pointed out that his game was zero-sum while Riker's and Maschler's games were nonzero-sum. Moreover, Lieberman argued that Riker:

> Used the same subjects repeatedly and permitted or encouraged them to discuss the task, its solution, and resolution outside of the experimental situation.... As Riker points out, this procedure has the effect of producing 'sophisticated' play and 'experienced' subjects (pp. 115–116).

Lieberman's counterclaim is also consistent with our assumption that experienced, sophisticated bargainers are more likely to reach an agreement on the late (asymptotic) rounds of negotiations. Thus the claims made by both Riker and Lieberman are consistent with our assumption regarding situational factors and rounds to reach agreement.

Evaluation in superadditive games

For the case when $v(G) > 0$, we shall use the data reported by Rapoport and Kahan (1976). Subjects in their study were volunteers (undergraduate students), and were paid five cents per point as an incentive to maximize in the games. Each subject gained an average of $25.00 (in three-hour sessions once a week for three weeks). Subjects were placed in separate rooms and bargaining was conducted by teletypewriters. Hence, in accordance with our assumption that rounds" represent a parameter of the model, we shall use the asymptote as an estimate of reward division.

Five three-person games were used and Table 5 shows the values of the four possible coalitions, the observed data, and the predictions of the model for the grand coalition. It can be seen that the asymptotic payoff predictions yield reasonably accurate estimates of the observed values. With regard to the frequency data, Table 5 shows that with the exception of game 5, the grand coalition occurred most frequently. Assuming that many rounds were required to reach agreement (supported by the payoff data), based on expectations on successive rounds of negotiations, it can be shown that both players B and C become relatively indifferent between the grand coalition and a coalition with player A. Assuming that players B and C initiate offers to form G, player A will be induced to reciprocate such offers to form G, and according to Assumption (3), the grand coalition should be likely to occur. This prediction is supported by the fact that over four iterations of each game, the frequencies of the grand coalition increased from 55% on the first iteration to 80% on the fourth iteration, strongly suggesting that a group norm had developed to form the grand coalition.

This prediction, however, is inconsistent with the frequency data for game 5, in which the AB coalition was most frequent. Rapoport and Kahan (1976) note that the difference between $v(ABC)$ and $v(AB)$ is only 3 points for game 5 (cf. Table 5), while for the other four games, the differences are at least 18 points. Hence, they state that, "Player C is not in a position to add any significant amount to the rewards of the two players; his three points (15 cents) may well not represent a fair reward for the work of negotiation, and his efforts may have been negligible" (p. 263). Their explanation is quite plausible and suggests that other assumptions about the bargaining process will be necessary to make more precise predictions about coalition likelihood in superadditive games.

With regard to the observed payoff data, Rapoport and Kahan compared the predicted rewards of various theories and report that the predictions of the bargaining set were most accurate. Since the predictions of the bargaining set for these games coincide with the asymptotic values, these data also provide support for the proposed model. Rapoport and Kahan also analyzed the first offer of each game, the first entry into the acceptance stage of each game, and the corresponding first offers and first acceptances for the ultimately winning coalition, and report that the offers at successive stages were "more egalitarian in nature the earlier they occurred in the game, although the effects of the players' relative power was immediately manifested." They conclude that, "Games are characterized by a progression towards the $M_1^{(i)}$ solution," (p. 267), where $M_1^{(i)}$ solution refers to the bargaining set predictions. Thus, the model is supported not only for its asymptotic predictions but also receives some support for the sequential changes in expectations over rounds.

Evaluation in simple games

In a study by Selten and Schuster (1968), the following five-person, simple game was

TABLE 5

MEAN DIVISION OF REWARDS AND MEAN PROPORTION OF OCCURRENCE, p(ABC), OF THE GRAND COALITION AND PREDICTED REWARDS IN THREE-PERSON GAMES.*

Game	Value of coalitions				Observed data		Asymptotic prediction**
	$v(AB)$	$v(AC)$	$v(BC)$	$v(ABC)$	$p(ABC)$	division	
1	95	90	65	120	.72	55–37–28	58–33–28
2	115	90	85	140	.75	60–52–28	58–53–28
3	95	88	81	127	.88	52–43–32	49–42–36
4	106	86	66	124	.78	57–44–23	61–41–21
5	118	84	50	121	.34†	68–45–8	74–40–6

* Data from Rapoport and Kahan (1976).

** The asymptotic predicted values are identical to the predictions of Aumann and Maschler's bargaining set.

† The AB coalition was most frequent in game 5, with $p(AB)$ = .56, and a mean reward division of 71-47 for A and B, respectively.

used: $v(AX) = v(AXX) = v(AXXX) = v(XXXX) = 40$; $v(\text{all others}) = 0$. In this game, there is a single "strong" player (A) and four "weaker" players (X). The prize to be divided was 40 deutsch marks (about $10 at the time), but in evaluating the model we shall translate deutsch marks into proportional shares of the prize. There were 12 groups and each group negotiated in a face-to-face situation, for a single trial of the game. Based on Eq. (1) and (2), it can be shown that the strong-weak alliance (AX) is predicted to be most likely for a 67-33 split on round 1, and a 75-25 split at the asymptote. Since subjects were paid volunteers and played for relatively large stakes, we shall use the asymptotic payoff predictions to evaluate the model.

The AX coalition occurred in 8 of 12 groups, the weak-union ($XXXX$) occurred twice, and the $AXXX$ and the grand coalitions each occurred once. Thus, the frequency data support the predictions of the model. However, the mean reward division for the AX coalition was 61-39 for A and X, respectively, and the payoff for player A is considerably less than those predicted by the model (67 on round 1 and 75 at the asymptote).

A plausible explanation of player A's low payoff is that in a face-to-face situation, the weaker players were aware of offers and counteroffers among all players, and soon realized that it was to their collective advantage to form the weak-union against player A (rather than compete against each other). Hence, player A may have been inhibited from increasing his demands over rounds, so as to tempt one of them to defect from the commitment to the weak-union. This hypothesis is consistent with the results of a study by Murnighan and Roth (1977), who varied communication and information among the players in a coalition experiment. Their results showed that, "the ability to communicate helped foster greater cooperation between the weaker players" (p. 1344).

Thus, if communication among the players had been restricted in some way, it is plausible that the payoffs for player A would not only have been greater, but the frequency of the AX coalition would also have been greater. This hypothesis is consistent with the results reported by Horowitz and Rapoport (1974), who used the same five-person game. In their study, subjects communicated by teletypewriter and the strong-weak alliance (AX) occurred in 90% of the cases. Moreover, player A's mean payoff in the AX coalition was much greater than in Selten and Schuster's study (72-29 split), and much closer to the asymptotic predicted value of a 75-25 split.

What is suggested here is an interaction between situational variables and the structure of the game, such that in a game where one person has a tremendous advantage over the other players, a face-to-face situation may make it easier to coordinate and "organize" the weak-union, making it difficult for the "strong" person to achieve the asymptotic payoff. Thus, a face-to-face situation may reduce the power of the "strong" person, while restrictions on communication and information may enhance his power.

SUMMARY AND CONCLUSION

One of the basic assumptions of the equal excess model is that coalition behavior is a function of expectations and that changes in expectations is a function of the quality of offers a person receives over successive rounds of negotiations. Another important assumption is that the likelihood of a given coalition is a function of the pattern of demands made by the players, and such demands are also based on changing expectations. Hence, if a given player does not make appropriate concessions over rounds, he may induce the others to reciprocate offers with each other, and he may be excluded in the early stages of negotiations. However, if he makes appropriate concessions (as predicted by the model) so as to disrupt the potential coalitions which exclude him, then at the asymptote all players should be indifferent between their respective alternative coalitions. In this case, the group's prior history of reciprocating offers (Assumption 3) will probably be an important factor that determines which coalition is likely to form.

Therefore the model is a "dynamic equilibrium model" in that expectations are assumed to be at its maximum disequilibrium in the early stage of negotiations, and gradually approach an equilibrium state at the asymptote. The strongest evidence for such changes in expectations are the results of the study by Rapoport and Kahan (1976), who found that the predictions of the bargaining set gradually became more accurate over successive iterations of their games, suggesting that payoffs were indeed converging to the asymptotic solution predicted by the model.

In evaluating the proposed model there were several discrepancies between predicted and observed values, but at this stage of theoretical development the model seems quite promising, especially because there is no other theory that predicts both coalition formation and reward division in a variety of coalition situations. It can be shown that the model makes reasonably accurate predictions for several studies that were not included, but because of space limitations, we have selected only a sample of studies to compare and review.

It is appropriate at this point to discuss the boundary conditions of the proposed model. First, a class of games called "pairwise quota games" (Shapley, 1953; Maschler, 1978) suggests that the model may not be applicable in games in which two or more disjoint coalitions have positive value. In the four-person case, various pairs of two-person coalitions can form, and the problem is to predict which pair—and in what sequence—they are likely to form. This type of game leads to difficulties for the model, and suggests an important boundary condition of the model: It may be restricted to situations in which only a single coalition with positive value can simultaneously occur. In the case of simple games, this means that two or more "winning" coalitions cannot occur independently and simultaneously (an improper game). In addition, in its present form predictions cannot be derived for studies in which the "power" of a player is manipulated by the assignment of "resources." In such games, there need not be a one-to-one correspondence between resource assignment and the strategic advantages of the players, and since such resources seem to have an effect on coalition behavior (cf. reviews by Gamson, 1964; Stryker, 1972), additional assumptions will be required to derive predictions for such situations.

For situations based on resource assignment, one possible extension of the model is to incorporate one of the assumptions of the bargaining theory (Komorita & Chertkoff, 1973). Such an extension assumes that the initial expectations of the players are based on two norms of reward division, equality and parity, and on the first round of negotiations (E^1_{iS}), the most likely reward division is based on splitting the difference between the two norms. Predictions on subsequent rounds would be based on iterations of Eq. (2).

With regard to the weaknesses of the model, one of the main problems is the prediction of the round on which an agreement is likely to be reached. Strictly speaking, in its present form all that can be predicted about reward division is that they should range between round 1 and the asymptote. It would be desirable to specify more precisely the conditions under which a given round is likely to yield the best estimate of reward division. In specifying such conditions, we have stressed individual differences in the competitive motivation of subjects, but there are reasons to believe that bargaining skill, familiarity, and experience with the structure of coalition games may be equally important. It is plausible that naive bargainers, when assigned to the "strong" (advantageous) position in a game, do not exploit their advantage to the fullest extent. This hypothesis is consistent with the discrepancy in results obtained by Riker (1967) and by Lieberman (1962), discussed earlier, and is consistent with the changes in payoffs over successive iterations of the games reported by Rapoport and Kahan (1976). Thus, the round that yields the best estimate of reward division may depend on many factors: the complexity of the game, the length and clarity of instructions, the number of practice trials (if any), time limits and other pressures to reach agreement, and the background and experience of subjects in real-life bargaining situations.

The problem of the effects of situational variables is not unique to the proposed model, but is a problem for any theory that does not directly take into account (postulate) the effects of situational factors on coalition behavior. Unfortunately, most investigators, with a few notable exceptions (cf. Gamson, 1964; Riker, 1971; Stryker, 1972), have paid little attention to possible situational variables that might differentially affect the validity of various theories. Gamson (1964), after reviewing the empirical literature, concluded that each of the theories he evaluated received some support—depending on the conditions under which the theory was tested. Hence, Gamson implies that some theories may be more valid than others in some situations but less valid in other situations. Since the proposed model does not specify the round on which an agreement is likely to occur, this weakness of the model may be its greatest strength: Efforts to correct this weakness may facilitate a systematic examination of the effects of such situational factors on coalition behavior.

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