An Equation for the Emergent Capacity of Artificial Life

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and

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Abstract

The concept of emergent capacity is introduced as a measure of a system's ability to support emergent behavior. An equation for emergent capacity is introduced and it is shown that this equation predicts the critical Lambda values at which 1-D cellular automata exhibit maximum emergent behavior. The critical Lambda value is related in a simple way to N, the number of neighbors used in the state transition mapping function. It is conjectured that the equation makes a fundamental statement about emergent systems in general and about artificial life in particular.

λ_c: The Critical Lambda Value

Langton invented the lambda parameter as a way to explore the enormous search space of cellular automata (CA) and found robust structure, artificial life, emerges near a critical value, λ_c (Langton 1986, 1990), (Wolfram 1984). This value lies at the phase transition point between order and chaos and analysis of this phenomenon gave rise to the now famous "life at the edge of chaos" mantra. However, while it is generally agreed that λ_c lies at some intermediate value, it has not yet been possible to predict that value by mathematical means. This paper introduces an equation making such prediction possible and in the process makes a fundamental statement about emergent systems in general.

Emergent Capacity

The notion of *emergent capacity* is introduced here and defined as a measure of the ability of a system to support emergent behavior (Fulbright 2002). Systems with large emergent capacities exhibit a great deal of emergent behavior. 1-D CAs, like those used in artificial life research, are highly emergent when their lambda parameters are tuned near the critical value, λ_c. This happens because the emergent capacity of the CA is maximized at this value. Emergence arises from the complex interactions among the internal components of the system. Emergent capacity then is a measure of how well a system supports such complex interactions. It is important to note that emergent capacity does not attempt to measure the quantity or quality of what emerges from a system, but rather indicates how much *can* emerge. In (Fulbright 2002), the general form of the equation for emergent capacity, as inspired by (Stonier 1990) is

$$I = me^{\eta(\log_2 \frac{1}{\eta})^c}, \qquad (1)$$

where m is a scaling factor equal to the emergent capacity of the system at zero-entropy, η is the normalized entropy metric (Shannon 1948) giving the relative complexity of the internal structure of the system, and c is a measure of fidelity used in the model of the system called the *coefficient of abstraction*.

The Abstraction Radius

In CA studies, one must specify the number of neighbors, N, and the number of states per cell, K yielding N^K possible states for each automaton in the cellular lattice. Here, we define the abstraction radius, r, shown in Figure 1 (Fulbright 2002).

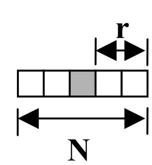

Figure 1 – Abstraction Radius of 1-D Cellular Automata

The shaded cell is the 1-D automaton and r adjacent cells on either side of the automaton form N neighbors. Mathematically, this is stated as

$$r = \frac{N-1}{2}. \qquad (2)$$

In (Fulbright 2002), the abstraction radius is related to the coefficient of abstraction used in equation (1) by

$$c = \ln(1 + r)\,. \tag{3}$$

Emergent Capacity of Cellular Automata

For artificial life studies, the lambda parameter is equivalent to η, and *m* can be set to unity if we remember that the result will be a relative quantity in *bits* per unit. This allows us to write equation (1) in the form

$$I = e^{\lambda(\log_2 \frac{1}{\lambda})^c}\,, \tag{4}$$

where λ is Langton's lambda parameter ranging from 0 to 1 with uniform behavior at values near 0 and random behavior at values near 1. The critical lambda value is the value at which this equation is maximized and is given by

$$\lambda_c = e^{-c}. \tag{5}$$

Substituting equation (3) for *c* yields the stunningly simple relation

$$\lambda_c = \frac{1}{1+r} = \frac{2}{1+N}\,, \tag{6}$$

in terms of *r* or *N* as desired. Varying the number of neighbors, equation (6) gives predictions for λ_c as shown in Table 1.

N	r	λ_c
1	0	1.000
3	1	0.500
5	2	0.333
7	3	0.250
9	4	0.200
11	5	0.167
13	6	0.143
15	7	0.125

Table 1 – Theoretical Values for λ_c

These are the values of λ at which the CA exhibits maximum emergent capacity. Consequently, these are the values at which robust, emergent structure should emerge.

Empirical Results

To test the theoretical predictions against empirical results, we use the artificial life applet shown in Figure 2 and located on the Internet at the address:

http://math.hws.edu/xJava/CA/EdgeOfChaosCA.html

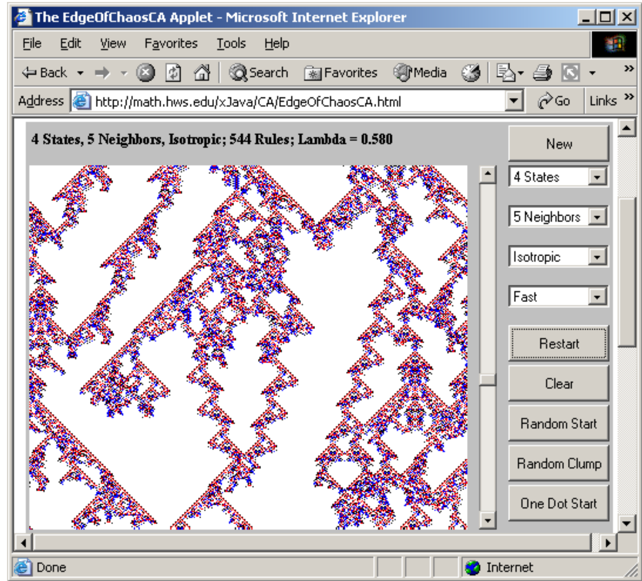

Figure 2 – The EdgeOfChaos Applet

This applet allows the user to set the number of neighbors, *N*, and the number of states per cell, *K*, then vary the value of lambda from 0 to 1 using the scroll bar. Generations of the CA paint from the top of the screen to the bottom.

Twelve settings of *N* and *K* were explored: (3,3) (3,4) (3,5) (3,6) (3,7) (5,3) (5,4) (5,5) (5,6) (5,7) (7,3) (7,4). A "run" consisted of stepping lambda through the entire range and tabulating the values at which emergent structure was observed. A total of 25 runs were made for each setting of *N* and *K*.

Two important characteristics were noted during the course of these experiments:

- Many runs exhibited more than one eruption of emergent structure at different lambda values.
- On any particular run, it was not possible to predict a priori which values of lambda would yield emergent structure.

Therefore, it was necessary to analyze the behavior in statistical terms. Each eruption of emergent structure in a run was tabulated as an "emergent point" by noting the lambda value. For each run, the emergent points were averaged to yield the average emergent point for that run. After completing all 25 runs, all average emergent points were averaged yielding a total average emergent point for the setting.

The results of the empirical study are shown in Table 2 and a plot of these results is shown in Figure 3 showing a dependence on *N*.

K	N	Avg. Emergent Pt.
3	3	0.646
4	3	0.510
5	3	0.488
6	3	0.526
7	3	0.441
3	5	0.284
4	5	0.331
5	5	0.295
6	5	0.301
7	5	0.299
3	7	0.259
4	7	0.261
		0.387

Table 2 – Empirical Results

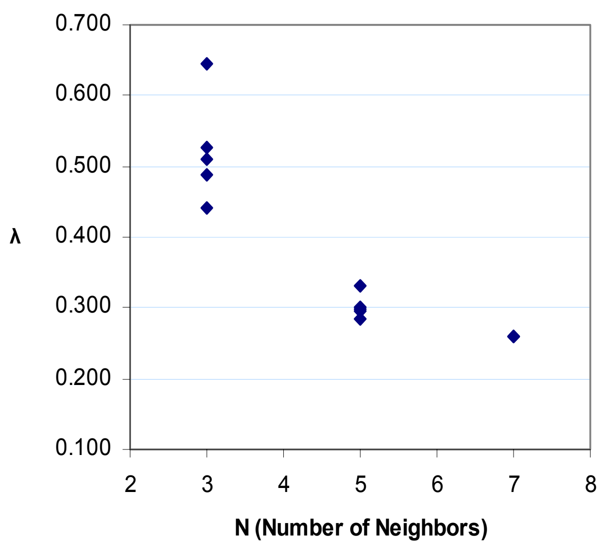

Figure 3 – Average Emergent Points

The plot is divided into three regimes (N=3, 5, 7). The average emergent point of each regime is shown in Table 3.

K	N	Regime Avg.
*	3	0.522
*	5	0.303
*	7	0.260

Table 3 – Average of the Regimes

These empirical results match the values predicted by equation (1) for N=3, N=5, and N=7 and shown in Table 1. Using equation (2), the horizontal axis in Figure 3 can be changed to *r* instead of *N*. Figure 4 shows the regime averages plotted against *r*.

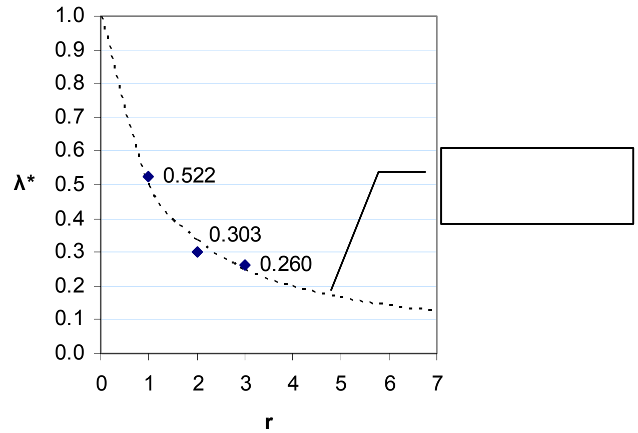

Figure 4 – Average Emergent Points

This is a convenient way of representing the results since, by using regression analysis, the simple curve

$$\lambda^* = \frac{1}{1+r} \qquad (7)$$

was found to fit the data points where λ^* is the average critical lambda value for the regime and *r* is the abstraction radius. This equation is equivalent to equation (6).

The Importance of Distance

Equation (1) calculates the emergent capacity of a system and includes two key terms: the relative entropy of the system, η, and the coefficient of abstraction, *c*. The idea that emergent behavior is dependent on relative entropy is well known to researchers in the artificial life field. However, considering the importance of the observer's *distance* is a new notion.

To an observer watching a flock of birds from a great distance, the flock appears as nothing more than a dot in the sky. The observer has no way of distinguishing the flock from any number of other possible objects. Likewise, viewing the flock from a distance of only a few feet allows the observer to see only a single bird and miss the flock altogether. This scenario illustrates the importance of physical distance when observing global, emergent behavior of a system.

However, there are other types of distance. An observer sensing the flock with inappropriate instrumentation might miss emergent flock behavior even if he or she were at the ideal physical distance.

Researchers constructing mathematical models of real-world systems, create abstract models of the system. No model has fidelity of 100%, so the level of detail in the model is another type of *distance*.

CA's are abstract models. The number of neighbors parameter, N is adjustable allowing one to probe artificial life with models of differing grain size. By varying r we are varying our *distance* from the system. Through the relationship between r and c, equation (1) tells us to expect a different emergent capacity when r is changed—a result verified empirically.

Conclusion

The concept of emergent capacity was introduced and defined as a measure of a system's ability to support emergent behavior. The idea was introduced that emergent capacity is dependent on the distance from which the system is observed. This is a fundamental notion whose ramifications are the subject of future work. The general form of the emergent capacity equation was introduced and specialized for use with cellular automata. The equation was used to predict, statistically, the critical Lambda values at which 1-D cellular automata exhibit maximum emergent behavior and these predictions were verified by empirical study.

References

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