Available online at www.sciencedirect.com
ScienceDirect
Artificial Intelligence 171 (2007) 675–700
Artificial Intelligence
www.elsevier.com/locate/artint

On principle-based evaluation of extension-based argumentation semantics ☆

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Received 13 November 2006; received in revised form 3 April 2007; accepted 16 April 2007
Available online 29 April 2007

Abstract

The increasing variety of semantics proposed in the context of Dung's theory of argumentation makes more and more inadequate the example-based approach commonly adopted for evaluating and comparing different semantics. To fill this gap, this paper provides two main contributions. First, a set of general criteria for semantics evaluation is introduced by proposing a formal counterpart to several intuitive notions related to the concepts of maximality, defense, directionality, and skepticism. Then, the proposed criteria are applied in a systematic way to a representative set of argumentation semantics available in the literature, namely grounded, complete, preferred, stable, semi-stable, ideal, prudent, and *CF*2 semantics.

Keywords: Argumentation frameworks; Argumentation semantics; Skepticism

1. Introduction

Interest and vitality of AI-related research in argumentation theory are witnessed, among other factors, by the increasingly rich variety of argumentation semantics being proposed in the literature. Though most of them share the influential theory of Dung [14] as a common reference framework, comparing and assessing semantics proposals appears far from being a straightforward task. In fact, various kinds of motivations have been used to support the introduction of new semantics with respect to "classical" proposals such as *stable* [23], *grounded* [19], *complete* [14] and *preferred* [14] semantics. These motivations range from the desire to formalize some high-level intuition, not captured by other proposals, to the need to achieve the "correct" treatment of a particular example (or family of examples), regarded as particularly significant. For instance, *CF*2 semantics [2,6] has originally been conceived to deal with some problematic behaviors of preferred semantics when dealing with odd-length cycles. Semi-stable semantics [9] aims at avoiding the problem of non-existence of extensions affecting stable semantics in some cases, while preserving its behavior when stable extensions exist as well as the related property of minimizing undecided arguments. Prudent

☆ This article is a substantial extension of the conference paper [P. Baroni, M. Giacomin, Evaluation and comparison criteria for extension-based argumentation semantics, in: P.E. Dunne, T. Bench-Capon (Eds.), Proceedings of the 1st International Conference on Computational Models of Arguments (COMMA 2006), IOS Press, Liverpool, UK, 2006, pp. 157–168].

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0004-3702/$ – see front matter
doi:10.1016/j.artint.2007.04.004

semantics [12,13] emphasizes the role of indirect attacks, which are not allowed within its extensions, while ideal semantics [15] provides a unique-status approach which is as computationally efficient as grounded semantics while being less skeptical.

Clearly, these kinds of heterogeneous intuitions hardly lend themselves to systematic comparisons. As a further difficulty, general comparison and evaluation criteria are actually lacking, since the problem of identifying and formally characterizing such criteria seems to have received relatively limited attention in the literature. Given this situation, it is not surprising that comparisons are quite often carried out using specific problematic examples, often ingeniously devised so as to bring to light patently different behaviors exhibited by the semantics under discussion. A sample collection of this kind of "benchmark problems" can be found for instance in [25]. While carefully selected examples can surely provide useful insights for the analysis of alternative semantics proposals, example-based comparisons suffer from several significant drawbacks. First of all, they are affected by the inherent limitation of relying more on intuition than on formally stated principles. In fact, even in relatively simple examples there may not be a general agreement on the "desired" outcome, due to different underlying intuitions (see for instance [16]). For this reason, it has been observed that using intuition about specific examples to derive general considerations may be inappropriate and "it is better to use intuitions not as critical tests but as generators for further investigation" [20]. The significance of benchmark problems as testbeds for evaluation and guidelines for semantics design is also put in question by Vreeswijk's interpolation theorem [25], which proves that "for any compatible collection of instantiated benchmark problems there exists an abstract argumentation system that complies with every instantiated benchmark problem". This suggests that "the enterprise of solving more and more of these problems does not necessarily bring us to the ultimate non-monotonic logic" [25] (see also [8, Chapter 2] for a discussion of this point). It has also to be remarked that some examples, explicitly conceived for one-to-one comparison of specific semantics, may turn out to be less significant when evaluating a large range of proposals. Moreover, these examples typically are aimed at evidencing (and possibly overemphasizing) differences, while a balanced evaluation should also take into account similarities and common features. Emphasis on disagreement gives rise, as a side-effect, to the quest for the "right" semantics, which, in our opinion, is a misleading objective. In fact, different application domains may well require different styles of reasoning, reflected by different semantics flavors. Characterizing the reasoning requirements of different domains (still a largely open research problem) can only be based on general criteria rather than on specific cases.

In this paper we investigate the definition of some general criteria for evaluating extension-based argumentation semantics in the framework of Dung's theory. The introduced criteria are then applied to several literature proposals, thus providing a principle-based systematic assessment of state-of-the-art approaches in argumentation semantics.

The paper is organized as follows. After recalling the necessary background concepts in Section 2, we define and discuss general criteria for evaluating sets of extensions in Section 3 and comparison criteria based on the notion of skepticism in Section 4. The definitions of grounded, complete, preferred, stable, semi-stable, ideal, prudent, and *CF*2 semantics are reviewed in Section 5: the introduced criteria are then applied to the above mentioned semantics in Section 6. Finally, a discussion of the relationships with related works and some conclusions are provided in Section 7.

2. Basic concepts

2.1. Abstract argumentation frameworks

Our work adopts as a reference Dung's theory of abstract argumentation frameworks.

Definition 1. An argumentation framework is a pair $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, where $\mathcal{A}$ is a set and $\rightarrow \subseteq (\mathcal{A} \times \mathcal{A})$ is a binary relation on $\mathcal{A}$, called attack relation.

We assume that $\mathcal{A}$ represents the set of arguments produced by a reasoner at a given instant of time, therefore in the following we will assume that $\mathcal{A}$ is finite, independently of the fact that the underlying mechanism of argument generation admits the existence of infinite sets of arguments. The treatment of argumentation frameworks where $\mathcal{A}$ is infinite, including the special case of *finitary* argumentation frameworks [14] where the number of attackers of each argument is finite, is left for future work.

An argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ can be represented as a directed graph, called the *defeat graph*, where nodes are the arguments and edges correspond to the elements of the attack relation. The nodes that attack a given argument α are called *defeaters* (or *parents*) of α and form a set which is denoted as $\mathrm{par}_{\mathrm{AF}}(\alpha)$.

Definition 2. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and an argument $\alpha \in \mathcal{A}$, $\mathrm{par}_{\mathrm{AF}}(\alpha) \triangleq \{\beta \in \mathcal{A} \mid \beta \rightarrow \alpha\}$. If $\mathrm{par}_{\mathrm{AF}}(\alpha) = \emptyset$, then α is called an *initial* argument. The set of initial arguments of AF is denoted as $IN(\mathrm{AF})$.

Since we will frequently consider properties of sets of arguments, it is useful to define suitable notations for them.

Definition 3. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, an argument $\alpha \in \mathcal{A}$ and two (not necessarily disjoint) sets $S, P \subseteq \mathcal{A}$, we define:

$$S \rightarrow \alpha \equiv \exists \beta \in S\colon\ \beta \rightarrow \alpha$$

$$\alpha \rightarrow S \equiv \exists \beta \in S\colon\ \alpha \rightarrow \beta$$

$$S \rightarrow P \equiv \exists \alpha \in S, \beta \in P\colon\ \alpha \rightarrow \beta$$

$$\mathrm{outpar}_{\mathrm{AF}}(S) = \{\alpha \in \mathcal{A} \mid \alpha \notin S \wedge \alpha \rightarrow S\}$$

$$\mathrm{outchild}_{\mathrm{AF}}(S) = \{\alpha \in \mathcal{A} \mid \alpha \notin S \wedge S \rightarrow \alpha\}$$

We also define the *restriction* of an argumentation framework to a subset S of its arguments.

Definition 4. Let $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ be an argumentation framework. The *restriction* of AF to $S \subseteq \mathcal{A}$ is the argumentation framework $\mathrm{AF}{\downarrow_S} = \langle S, \rightarrow \cap (S \times S) \rangle$.

2.2. Extension-based argumentation semantics

Given an argumentation framework encoding the conflicts among a set of arguments, a fundamental problem consists in determining the conflict outcome, namely which arguments can be considered justified. An argumentation semantics can be conceived, in broad terms, as a formal way to answer this question. In argumentation literature, two main styles of semantics definition can be identified: *extension-based* and *labeling-based*.

An *extension-based* argumentation semantics is defined by specifying the criteria for deriving, for a generic argumentation framework, a set of *extensions*, where each extension represents a set of arguments considered to be acceptable together. Given a generic argumentation semantics $\mathcal{S}$, the set of extensions prescribed by $\mathcal{S}$ for a given argumentation framework AF is denoted as $\mathcal{E}_{\mathcal{S}}(\mathrm{AF})$. If a semantics $\mathcal{S}$ prescribes $\mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ to be a singleton for any argumentation framework AF, $\mathcal{S}$ is said to belong to the *unique-status approach*, otherwise to the *multiple-status approach* [22].

A *labeling-based* argumentation semantics relies on a predefined set of labels and is defined by specifying the criteria for assigning them to the arguments of a generic argumentation framework. Depending on the criteria, one or (typically) more possible labellings are obtained. Most argumentation semantics proposed in the literature are extension-based (e.g. grounded, complete, stable, preferred, semi-stable, ideal, prudent, and *CF*2 semantics) while examples of labeling-based proposals are represented by robust semantics [17] and by argumentation stages [24]. It has also to be remarked that in some cases a labeling definition has an equivalent counterpart in terms of extensions (see for instance [10]). For these reasons, we focus on extension-based semantics and leave the consideration of labeling-based approaches for future work.

The extensions prescribed by a semantics are the basis for deriving the *justification state* of each argument. Though a more articulated treatment is possible [5], in this paper we consider a basic classification encompassing just two possible states for an argument, namely justified or not justified. Two alternative types of justification, namely *skeptical* and *credulous* can be considered.

Definition 5. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and a semantics $\mathcal{S}$, an argument α is *skeptically justified* iff $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})\ \alpha \in E$, and is *credulously justified* iff $\exists E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})\colon\ \alpha \in E$.

A relevant question concerns the existence of extensions. While the definition of most semantics ensures that there is at least one extension for any argumentation framework, it is well known that this is not always the case. In particular, there are defeat graphs including odd-length cycles where stable semantics fails to provide any extension. To settle this difficulty once and for all, we take as a standpoint that argumentation frameworks for which no extension exists are simply outside the applicability domain of the considered semantics and therefore they neither play any role in the properties of the semantics nor can the semantics be used in any way to assess the justification of their arguments. Formally, for a generic semantics $\mathcal{S}$ let $\mathcal{D}_{\mathcal{S}}$ be the set of argumentation frameworks where $\mathcal{S}$ admits at least one extension, namely $\mathcal{D}_{\mathcal{S}} = \{\mathrm{AF}\colon \mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \neq \emptyset\}$: in the following, whenever we will refer to the properties of a semantics $\mathcal{S}$ for an argumentation framework AF we will implicitly assume that $\mathrm{AF} \in \mathcal{D}_{\mathcal{S}}$ if not differently specified. Note that the case of non-existence of extensions, namely $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \emptyset$, is significantly different from the case $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \{\emptyset\}$, where the semantics prescribes exactly one (actually empty) extension. Note in particular that, in the latter case, $\mathrm{AF} \in \mathcal{D}_{\mathcal{S}}$.

Let us now introduce two fundamental principles underlying the definition of extension-based semantics in Dung's framework, namely the *language independence* principle and the *conflict free* principle.

The *language independence* principle intuitively consists in the fact that the extensions prescribed by a semantics only depend on the relations of attack between arguments considered as abstract entities, namely on the topology of the corresponding defeat graph, while extensions are totally independent of any property of arguments at the underlying level of the language where their internal structure and features can be explicitly represented. Formally, this principle corresponds to the fact that argumentation frameworks which are isomorphic have the "same" (modulo the isomorphism) extensions, as stated by the following definitions.

Definition 6. Two argumentation frameworks $\mathrm{AF}_1 = \langle \mathcal{A}_1, \rightarrow_1 \rangle$ and $\mathrm{AF}_2 = \langle \mathcal{A}_2, \rightarrow_2 \rangle$ are isomorphic if and only if there is a bijective mapping $m : \mathcal{A}_1 \rightarrow \mathcal{A}_2$, such that $(\alpha, \beta) \in \rightarrow_1$ if and only if $(m(\alpha), m(\beta)) \in \rightarrow_2$. This is denoted as $\mathrm{AF}_1 \stackrel{\circ}{=}_m \mathrm{AF}_2$.

Definition 7. A semantics $\mathcal{S}$ satisfies the language independence principle if and only if $\forall\, \mathrm{AF}_1 = \langle \mathcal{A}_1, \rightarrow_1 \rangle, \forall\, \mathrm{AF}_2 = \langle \mathcal{A}_2, \rightarrow_2 \rangle\colon\ \mathrm{AF}_1 \stackrel{\circ}{=}_m \mathrm{AF}_2, \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_2) = \{M(E) \mid E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_1)\}$, where $M(E) = \{\beta \in \mathcal{A}_2 \mid \exists \alpha \in E, \beta = m(\alpha)\}$.

All semantics defined at the abstract level of Dung's framework, and in particular all semantics we will deal with in this paper, adhere to the language independence principle.

The *conflict free* principle, denoted as $\mathcal{CF}$, intuitively relies on the idea that, since an extension is a set of arguments which are "acceptable together", no conflicting arguments can be included in the same extension. The $\mathcal{CF}$ principle has a straightforward formal counterpart.

Definition 8. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $S \subseteq \mathcal{A}$ is *conflict-free*, denoted as $cf(S)$, iff $\nexists \alpha, \beta \in S$ such that $\alpha \rightarrow \beta$. A semantics $\mathcal{S}$ satisfies the $\mathcal{CF}$ *principle* if and only if $\forall\, \mathrm{AF}, \forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ E is conflict-free.

As to our knowledge, all extension-based argumentation semantics proposed in the literature[1] adhere to the $\mathcal{CF}$ principle, which in the following will be given for granted. Therefore, where no further specifications are given, by a set of extensions of an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ we will mean a set of conflict-free subsets of $\mathcal{A}$.

3. Extension evaluation criteria

By an extension evaluation criterion we mean a property of a set of extensions on its own, not involving a comparison with other sets of extensions. We discuss evaluation criteria related to three different aspects: the maximality of extensions, the notion of defense, and the topology of the defeat graph.

[1] Note, however, that the definition of the $\mathcal{CF}$ principle as given in this paper is not directly applicable to literature approaches featuring a richer structure of argumentation frameworks and lying therefore outside the strict context of Dung's theory (e.g. preference-based [1] or value-based [7] argumentation frameworks).

3.1. *The* I-*maximality criterion*

Given that an extension can be intuitively conceived as a set of arguments that can be accepted together according to some semantics-specific requirements, one may consider as an additional constraint that no extension can be a proper subset of another one. This has a straightforward formal counterpart.

Definition 9. A set of extensions $\mathcal{E}$ is I-maximal iff $\forall E_1, E_2 \in \mathcal{E}$, if $E_1 \subseteq E_2$ then $E_1 = E_2$. A semantics $\mathcal{S}$ satisfies the I-*maximality criterion* if and only if $\forall$ AF $\mathcal{E}_\mathcal{S}(\mathrm{AF})$ is I-maximal.

Note that I-maximality is a property of the set of extensions $\mathcal{E}$ *per se* and does not imply that maximality is prescribed by the semantics-specific definition of what an extension is. For instance any unique-status semantics necessarily satisfies I-maximality according to Definition 9, independently of the fact that the unique extension prescribed by the semantics is a maximal set in any sense. I-maximality is an important criterion in relation to the notion of skeptical justification: given two extensions E_1 and E_2 belonging to a set of extensions $\mathcal{E}$, if $E_1 \subsetneq E_2$ then any argument $\alpha \in E_2 \setminus E_1$ cannot be skeptically justified according to Definition 5. As a limit case, if the empty set is included in the set of extensions then any argument cannot be skeptically justified, independently of other extensions. To avoid this kind of undesired effect, I-maximality is a crucial criterion and is in fact enforced by most semantics, as it will be discussed in Section 6. On the other hand, when the set of extensions is not meant to be used for evaluating skeptical justification, I-maximality is not fundamental any more.

3.2. *Defense related criteria*

As the arguments belonging to an extension can be regarded as "surviving the conflict together", the notion of defense against attacks coming from other arguments has always played an important role in argumentation semantics. Formally, this has a counterpart in the requirement of *admissibility*, which lies at the heart of all semantics discussed in [14] and is based on the well-known notions of acceptable argument and admissible set.

Definition 10. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, an argument $\alpha \in \mathcal{A}$ is *acceptable* with respect to a set $E \subseteq \mathcal{A}$ if and only if $\forall \beta \in \mathcal{A}$: $\beta \rightarrow \alpha$, $E \rightarrow \beta$.

Definition 11. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $E \subseteq \mathcal{A}$ is *admissible* if and only if E is conflict-free and $\forall \beta \in \mathcal{A}$: $\beta \rightarrow E$, $E \rightarrow \beta$. The set made up of all the admissible sets of AF will be denoted as $\mathcal{AS}(\mathrm{AF})$.

Definition 12. A semantics $\mathcal{S}$ satisfies the *admissibility criterion* if $\forall \mathrm{AF} \in \mathcal{D}_\mathcal{S}$, $\forall E \in \mathcal{E}_\mathcal{S}(\mathrm{AF})$ $E \in \mathcal{AS}(\mathrm{AF})$, namely:

$$\alpha \in E \Rightarrow \forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)\ E \rightarrow \beta \tag{1}$$

Condition (1) includes the case where α defends itself against (some of) its defeaters. We suggest that a stronger notion of defense may also be considered where a node α can not defend itself nor can be involved in its own defense. To this purpose we introduce the notion of *strongly defended argument*.

Definition 13. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\alpha \in \mathcal{A}$ and $S \subseteq \mathcal{A}$, we say that α is strongly defended by S (denoted as $sd(\alpha, S)$) iff $\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)\ \exists \gamma \in S \setminus \{\alpha\}$: $\gamma \rightarrow \beta$ and $sd(\gamma, S \setminus \{\alpha\})$.

In words, α is strongly defended by S if S includes a defeater $\gamma \neq \alpha$ for any defeater β of α; in turn, γ has to be strongly defended by $S \setminus \alpha$, namely γ needs neither α nor itself to be defended against its defeaters in AF. The recursion is well founded since, at any step, a set of strictly lesser cardinality is considered. In particular, in case $sd(\alpha, S)$ the base of this recursive definition is provided by initial nodes, which are strongly defended by any set since they have no defeaters. The notion of strong defense is the basis for the definition of the *strong admissibility criterion*.

Definition 14. A semantics $\mathcal{S}$ satisfies the *strong admissibility criterion* if $\forall \mathrm{AF} \in \mathcal{D}_\mathcal{S}$, $\forall E \in \mathcal{E}_\mathcal{S}(\mathrm{AF})$ it holds that:

$$\alpha \in E \Rightarrow sd(\alpha, E) \tag{2}$$

The property of *reinstatement* corresponds to the converse of the implication (1) prescribed by the admissibility criterion. Intuitively, an argument α is *reinstated* if its defeaters are in turn defeated and, as a consequence, one may assume that they should have no effect on the justification state of α. Under this assumption, if an extension E reinstates α then α should belong to E. Formally, this leads to the following *reinstatement criterion*.

Definition 15. A semantics $\mathcal{S}$ satisfies the *reinstatement criterion* if $\forall\,\mathrm{AF} \in \mathcal{D}_{\mathcal{S}}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ it holds that:

$$\big(\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)\ E \to \beta\big) \Rightarrow \alpha \in E \tag{3}$$

Considering the strong notion of defense we obtain a *weak* (since it is implied by condition (3)) *reinstatement criterion*.

Definition 16. A semantics $\mathcal{S}$ satisfies the *weak reinstatement criterion* if $\forall\,\mathrm{AF} \in \mathcal{D}_{\mathcal{S}}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ it holds that:

$$sd(\alpha, E) \Rightarrow \alpha \in E \tag{4}$$

Another observation concerns the fact that condition (3) prescribes that an argument α defended by an extension should be included in the extension, without specifying that α should not give rise to conflicts within the extension. To explicitly take into account this aspect, the following $\mathcal{CF}$*-reinstatement criterion* can be given.

Definition 17. A semantics $\mathcal{S}$ satisfies the $\mathcal{CF}$*-reinstatement criterion* if $\forall\,\mathrm{AF} \in \mathcal{D}_{\mathcal{S}}$, $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ it holds that:

$$\Big(\big(\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)\ E \to \beta\big) \wedge cf\big(E \cup \{\alpha\}\big)\Big) \Rightarrow \alpha \in E \tag{5}$$

3.3. The directionality criterion

The notion of directionality in argumentation semantics has first been considered in [6], where a general scheme relating the topology of the defeat graph with the definition of extensions has been introduced. At an intuitive level, the idea is quite simple: the justification state of an argument α should be affected only by the justification state of the defeaters of α (which in turn are affected by their defeaters and so on), while the arguments which only receive an attack from α (and in turn those which are attacked by them and so on) should not have any effect on the state of α. We provide here an extended notion of directionality with respect to [6] by considering a set of arguments not receiving attacks from outside.

Definition 18. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \to \rangle$, a set $U \subseteq \mathcal{A}$ is *unattacked* if and only if $\nexists \alpha \in (\mathcal{A} \setminus U)$: $\alpha \to U$. The set of unattacked sets of AF is denoted as $\mathcal{US}(\mathrm{AF})$.

The directionality criterion can then be defined by requiring an unattacked set to be unaffected by the remaining parts of the argumentation framework as far as extensions are concerned.

Definition 19. A semantics $\mathcal{S}$ satisfies the directionality criterion if and only if $\forall\,\mathrm{AF} = \langle \mathcal{A}, \to \rangle$, $\forall U \in \mathcal{US}(\mathrm{AF})$, $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, U) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow_U})$, where $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, U) \triangleq \{(E \cap U) \mid E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})\} \subseteq 2^U$.

In words, the intersection of any extension prescribed by $\mathcal{S}$ for AF with an unattacked set U is equal to one of the extensions prescribed by $\mathcal{S}$ for the restriction of AF to U, and vice versa.

While the definition of directionality given here refers to a single argumentation framework, it can be easily reformulated as a criterion concerning distinct argumentation frameworks which are equal as far as their restriction to an unattacked set is concerned.

Proposition 20. *A semantics $\mathcal{S}$ satisfies the directionality criterion if and only if $\forall\,\mathrm{AF}_1 = \langle \mathcal{A}_1, \to_1 \rangle$, $\forall\,\mathrm{AF}_2 = \langle \mathcal{A}_2, \to_2 \rangle$ such that $\exists U \in (\mathcal{US}(\mathrm{AF}_1) \cap \mathcal{US}(\mathrm{AF}_2))$: $\mathrm{AF}_1{\downarrow_U} = \mathrm{AF}_2{\downarrow_U}$ it holds that*

$$\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}_1, U) = \mathcal{AE}_{\mathcal{S}}(\mathrm{AF}_2, U) \tag{6}$$

Proof. Definition 19 directly implies condition (6) since $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}_1, U) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_1{\downarrow_U}) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_2{\downarrow_U}) = \mathcal{AE}_{\mathcal{S}}(\mathrm{AF}_2, U)$. On the other hand, considering any argumentation framework AF and $U \in \mathcal{US}(\mathrm{AF})$ we can apply condition (6) with $\mathrm{AF}_1 = \mathrm{AF}$ and $\mathrm{AF}_2 = \mathrm{AF}{\downarrow_U}$, obtaining $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, U) = \mathcal{AE}_{\mathcal{S}}(\mathrm{AF}{\downarrow_U}, U) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow_U})$, since for any $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, \mathcal{A}) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$. □

4. Skepticism related criteria

The notion of skepticism has often been used in informal or semi-formal ways to discuss semantics behavior, e.g. by observing that a semantics is "more skeptical" than another one. Intuitively, a semantics is more skeptical than another if it makes less committed choices about the justification state of the arguments. Roughly, the most skeptical conceivable semantics does not make any commitment on any argument, leaving all of them undecided, while a semantics is not skeptical at all if it takes a definite position about every argument in any argumentation framework.

While the issue of skepticism has been considered so far in the literature mostly in the case of comparison between specific proposals, we are interested here in a more general analysis. We consider, as a first step, a generic relation of skepticism $\preceq^E$ between sets of extensions: given two sets of extensions $\mathcal{E}_1, \mathcal{E}_2$ of an argumentation framework AF, $\mathcal{E}_1 \preceq^E \mathcal{E}_2$ will simply denote that $\mathcal{E}_1$ is at least as skeptical as $\mathcal{E}_2$ in some sense. In the context of semantics evaluation, a skepticism relation $\preceq^E$ can be used to compare the sets of extensions prescribed by a particular semantics on different but related argumentation frameworks. To this purpose, one first needs to define a skepticism relation between argumentation frameworks based on the same set of arguments: given two argumentation frameworks $\mathrm{AF}_1 = \langle \mathcal{A}, \rightarrow_1 \rangle$ and $\mathrm{AF}_2 = \langle \mathcal{A}, \rightarrow_2 \rangle$, $\mathrm{AF}_1 \preceq^A \mathrm{AF}_2$ denotes that AF_1 (actually its attack relation) is inherently less committed (to be precise, not more committed) than AF_2. Then, one may reasonably require that any semantics reflects in its extensions the skepticism relations between argumentation frameworks. Requirements of this kind will be called *adequacy criteria*. On the other hand, a skepticism relation between sets of extensions might also be used to compare the behavior of different semantics when applied to the same argumentation framework. Comparison between semantics according to this kind of relations is not considered in this paper and is left for future work.

It is worth noting that the skepticism relations introduced above are not necessarily total orders, since in general there can be two sets of extensions (or two argumentation frameworks) which are not comparable. Having introduced skepticism relations at an abstract level, let us examine in the following subsections how they can be concretely defined.

4.1. Skepticism relations between sets of extensions

Let us start by noting that defining a relation of skepticism between two extensions is intuitively straightforward: an extension E_1 is more skeptical than another extension E_2 if and only if $E_1 \subseteq E_2$. In fact, a more skeptical attitude corresponds to a smaller set of arguments surviving the conflict. This is actually the relation that can be applied to compare the behavior of two distinct unique-status semantics, as it is done for ideal and grounded semantics in [15]. Directly extending the above intuition to the comparison of sets of extensions leads to define the following elementary skepticism relation $\preceq^E_\cap$.

Definition 21. Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework AF, $\mathcal{E}_1 \preceq^E_\cap \mathcal{E}_2$ iff

$$\bigcap_{E_1 \in \mathcal{E}_1} E_1 \subseteq \bigcap_{E_2 \in \mathcal{E}_2} E_2 \tag{7}$$

In words, condition (7) corresponds to the fact that arguments skeptically justified according to $\mathcal{E}_1$ are also skeptically justified according to $\mathcal{E}_2$, which appears to be an essential requirement and provides an intuitive bound for any relation of skepticism.

Relation $\preceq^E_\cap$ is based on considerations referring only to skeptically justified arguments. Finer (and actually stronger) skepticism relations can be defined by considering relationships of pairwise inclusion between extensions. As a starting point, we recall that to compare a single extension E_1 with a set of extensions $\mathcal{E}_2$, the relation $\forall E_2 \in \mathcal{E}_2$ $E_1 \subseteq E_2$ has often been used in the literature (for instance to verify that the unique extension prescribed by grounded semantics is more skeptical than the set of extensions prescribed by preferred semantics). A direct generalization to the comparison of two sets of extensions is represented by the following weak skepticism relation $\preceq^E_W$, first introduced by the authors in [5].

Definition 22. Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework AF, $\mathcal{E}_1 \preceq^E_W \mathcal{E}_2$ iff

$$\forall E_2 \in \mathcal{E}_2\ \exists E_1 \in \mathcal{E}_1\colon\ E_1 \subseteq E_2 \tag{8}$$

Relation $\preceq^E_W$ is unidirectional, since it only constrains the extensions of $\mathcal{E}_2$, while $\mathcal{E}_1$ may contain additional extensions unrelated to those of $\mathcal{E}_2$. One may then consider also a more symmetric (and stronger) relationship $\preceq^E_S$, where it is also required that any extension of $\mathcal{E}_1$ is included in an extension of $\mathcal{E}_2$.

Definition 23. Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework AF, $\mathcal{E}_1 \preceq^E_S \mathcal{E}_2$ iff $\mathcal{E}_1 \preceq^E_W \mathcal{E}_2$ and the following condition holds:

$$\forall E_1 \in \mathcal{E}_1\ \exists E_2 \in \mathcal{E}_2\colon\ E_1 \subseteq E_2 \tag{9}$$

Having introduced three alternative skepticism relations between sets of extensions, let us now analyze some of their properties. First, it is immediate to see that the three relations are in strict order of implication. Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ of an argumentation framework AF, it holds that:

$$\mathcal{E}_1 \preceq^E_S \mathcal{E}_2 \Rightarrow \mathcal{E}_1 \preceq^E_W \mathcal{E}_2 \Rightarrow \mathcal{E}_1 \preceq^E_\cap \mathcal{E}_2 \tag{10}$$

Then, it can be easily seen (proof is omitted,[2] see [5] for a related result) that the relations give rise to a preorder.

Proposition 24. *Relations $\preceq^E_\cap$, $\preceq^E_W$ and $\preceq^E_S$ are preorders, i.e. they are reflexive and transitive.*

A simple example reveals that $\preceq^E_S$ is not antisymmetric. In fact, let us consider three extensions E_1, E_2 and E_3 such that $E_1 \subsetneq E_2 \subsetneq E_3$, and let $\mathcal{E}_1$ be $\{E_1, E_2, E_3\}$, $\mathcal{E}_2$ be $\{E_1, E_3\}$. It can be seen that $\mathcal{E}_1 \preceq^E_S \mathcal{E}_2$ and $\mathcal{E}_2 \preceq^E_S \mathcal{E}_1$, however $\mathcal{E}_1 \neq \mathcal{E}_2$. Since $\preceq^E_S$ entails the other relations, none of them is antisymmetric and therefore none of the three relations is a partial order. It is immediate to show (proof is omitted) that if the additional constraint of I-maximality is introduced, then $\preceq^E_W$ (and as a consequence $\preceq^E_S$) turns out to be a partial order (while it is easy to see that $\preceq^E_\cap$ is anyway not).

Proposition 25. *Given two* I*-maximal sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$, if $\mathcal{E}_1 \preceq^E_W \mathcal{E}_2$ and $\mathcal{E}_2 \preceq^E_W \mathcal{E}_1$ then $\mathcal{E}_1 = \mathcal{E}_2$.*

4.2. A skepticism relation between argumentation frameworks

While we are not aware of any definition of a general skepticism relation for argumentation frameworks in the literature, some more specific but related notions have been introduced by the authors in [3] and by Modgil in [18]. The skepticism relation $\preceq^A$ we propose for argumentation frameworks provides a generalization of these notions, while relying on the same common intuition concerning the relationship between a mutual and a unidirectional attack involving two arguments α and β.

To illustrate the intuition discussed in [3], let us consider a very simple argumentation framework including just two arguments α and β, where α attacks β but not vice versa. This is a situation where the state assignment of any argumentation semantics corresponds to the maximum level of commitment: it is universally accepted that α should be justified and β rejected. Now if we consider a modified argumentation framework where an attack from β to α has been added, we obtain a situation where, clearly, a lesser level of commitment is appropriate: given the mutual attack between the two arguments, neither of them can be assigned a definitely committed state and we are left in a more undecided, i.e. more skeptical, situation in absence of any reason for preferring either of them. Extending this reasoning, consider a couple of arguments α and β in a generic argumentation framework AF′ such that $\alpha \to \beta$ while $\beta \not\to \alpha$. Consider now an argumentation framework AF obtained from AF′ by simply adding an

[2] To keep the size of the paper within reasonable limits, several proofs are omitted. They can be found in [4].

attack relation from β to α while leaving all the rest unchanged. It seems reasonable to state that AF corresponds to a more undecided situation with respect to AF′. In fact, converting a unidirectional attack into a mutual attack can only make the justification states of the involved nodes less committed (of course, they can remain the same if they are strictly determined by other arguments, independently of the attack relations between α and β). In turn, having α or β in a less committed state may only give rise to other less committed states in the nodes they attack: intuitively, the more undecided is the state of an attacker, the more undecided should be the state of the attacked node, and, in turn, of the nodes attacked by the latter and so on. This reasoning will be generalized here by considering any number of "transformations" of unidirectional attacks into mutual ones.

Dealing with the case of hierarchical argumentation frameworks but relying on the same intuition, Modgil has independently introduced in [18] the notion of *partial resolution* of an argumentation framework AF, which, roughly speaking, is an argumentation framework AF′ where some mutual attacks of AF are converted into unidirectional ones. AF′ is a (complete) *resolution* of AF if all mutual attacks of AF are converted. Also in [18] the underlying idea is that the presence of mutual attacks corresponds to a more undecided situation.

Given these bases, we are now ready to define a skepticism relation $\preceq^A$ between argumentation frameworks based on the same set of arguments, which requires a preliminary notation concerning the set of conflicting pairs of arguments in an argumentation framework.

Definition 26. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathcal{CONF}(\mathrm{AF}) \triangleq \{(\alpha, \beta) \in \mathcal{A} \times \mathcal{A} \mid \alpha \rightarrow \beta \vee \beta \rightarrow \alpha\}$.

Definition 27. Given two argumentation frameworks $\mathrm{AF}_1 = \langle \mathcal{A}, \rightarrow_1 \rangle$ and $\mathrm{AF}_2 = \langle \mathcal{A}, \rightarrow_2 \rangle$, $\mathrm{AF}_1 \preceq^A \mathrm{AF}_2$ if and only if $\mathcal{CONF}(\mathrm{AF}_1) = \mathcal{CONF}(\mathrm{AF}_2)$ and $\rightarrow_2 \subseteq \rightarrow_1$.

It is easy to see that the above definition covers all cases where some (possibly none) mutual attacks of AF_1 correspond to unidirectional attacks in AF_2, while unidirectional attacks of AF_1 are the same in AF_2 (using the terminology of [18], AF_2 is a partial resolution of AF_1).

It is immediate to see that $\preceq^A$ is a partial order, as it consists of an equality and a set inclusion relation. Comparable argumentation frameworks feature the same set of arguments and the same set of conflicting pairs of arguments, and can not include opposite unidirectional attacks between the same pair of arguments. It is also worth noting that within the set of argumentation frameworks comparable with a given AF there are, in general, several maximal elements with respect to $\preceq^A$, namely all argumentation frameworks where no mutual attack is present (corresponding to the notion of resolution in [18]). In the following we will denote as $\mathcal{RES}(\mathrm{AF})$ the set of argumentation frameworks comparable with AF and maximal with respect to $\preceq^A$.

4.3. Skepticism adequacy

Given an argumentation framework which is more skeptical than another one, it is reasonable to require that, when applying the same semantics to both, the skepticism relation between them is preserved between their sets of extensions. This kind of criterion, called *skepticism adequacy*, has first been proposed in [3] and is formulated here in a generalized version.

Definition 28. Given a skepticism relation $\preceq^E$ between sets of extensions, a semantics $\mathcal{S}$ is $\preceq^E$-skepticism-adequate, denoted as $\mathcal{SA}_{\preceq^E}(\mathcal{S})$, if and only if for any pair of argumentation frameworks AF, AF′ such that $\mathrm{AF} \preceq^A \mathrm{AF}'$ the following condition holds $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \preceq^E \mathcal{E}_{\mathcal{S}}(\mathrm{AF}')$.

According to the definitions provided in Section 4.1 we have three skepticism adequacy properties, related by the same order of implication as in (10):

$$\mathcal{SA}_{\preceq^E_S}(\mathcal{S}) \Rightarrow \mathcal{SA}_{\preceq^E_W}(\mathcal{S}) \Rightarrow \mathcal{SA}_{\preceq^E_\cap}(\mathcal{S}) \tag{11}$$

It is easy to see that Definition 28 is equivalent to the one given in [3] where the modification of just one attack relation is considered, provided that, as in our case, skepticism relations are transitive.

Proposition 29. *Given a transitive skepticism relation $\preceq^E$ between sets of extensions and a semantics $\mathcal{S}$, $\mathcal{SA}_{\preceq^E}(\mathcal{S})$ if and only if for any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *for any* $\alpha, \beta \in \mathcal{A}$: $\alpha \neq \beta \wedge \alpha \rightarrow \beta$,

$$\mathcal{E}_{\mathcal{S}}(\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}) \preceq^E \mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \tag{12}$$

where $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)} = \langle \mathcal{A}, \rightarrow \cup \{(\beta,\alpha)\} \rangle$.

Proof. Clearly $\preceq^E$-skepticism adequacy implies (12) as a special case. On the other hand, for any pair of argumentation frameworks AF, AF′ such that $\mathrm{AF} \preceq^A \mathrm{AF}'$, it is clearly possible to construct at least one sequence $\mathrm{AF}_1, \ldots, \mathrm{AF}_n$ such that $\mathrm{AF}_1 = \mathrm{AF}$, $\mathrm{AF}_n = \mathrm{AF}'$, and for $i = 1, \ldots, n-1$, $\exists \alpha_i, \beta_i \in \mathcal{A}$ such that $\mathrm{AF}_i = \mathrm{AF}_{i+1}{}^{\rightleftarrows(\alpha_i,\beta_i)}$. Then if (12) holds, we have $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_1) \preceq^E \cdots \preceq^E \mathcal{E}_{\mathcal{S}}(\mathrm{AF}_n) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}')$, and, by transitivity, $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \preceq^E \mathcal{E}_{\mathcal{S}}(\mathrm{AF}')$. □

4.4. Resolution adequacy

In the context of hierarchical argumentation frameworks, the following requirement for an argumentation semantics $\mathcal{S}$ is proposed in [18] for any argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and for any argument $\alpha \in \mathcal{A}$:

$$\alpha \in \bigcap_{E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})} E \Leftrightarrow \forall \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})\ \alpha \in \bigcap_{E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF}')} E \tag{13}$$

It can be seen that the $\Rightarrow$-part of (13) is a special case of the skepticism adequacy criterion of Definition 28, where, among the argumentation frameworks AF′ such that $\mathrm{AF} \preceq^A \mathrm{AF}'$, only the maximal ones are considered and $\preceq^E_\cap$ is selected as the skepticism relation $\preceq^E$ between sets of extensions.

On the other hand, the $\Leftarrow$-part of (13) represents an original criterion, relying on the intuition that if an argument is skeptically justified in all possible resolutions of an argumentation framework AF then it should be skeptically justified in AF too. We refer to this criterion as *resolution adequacy* and provide now a generalization of its original formulation, in order to make it parametric with respect to skepticism relations between sets of extensions. In fact, letting $\mathcal{UR}(\mathrm{AF}, \mathcal{S}) = \bigcup_{\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})} \mathcal{E}_{\mathcal{S}}(\mathrm{AF}')$, it can be seen that the $\Leftarrow$-part of (13) is equivalent to $\mathcal{UR}(\mathrm{AF}, \mathcal{S}) \preceq^E_\cap \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$. Generalizing this condition leads to a parametric formulation of the resolution adequacy criterion.

Definition 30. Given a skepticism relation $\preceq^E$ between sets of extensions, a semantics $\mathcal{S}$ is $\preceq^E$-resolution adequate, denoted $\mathcal{RA}_{\preceq^E}(\mathcal{S})$, if and only if for any argumentation framework AF it holds that $\mathcal{UR}(\mathrm{AF}, \mathcal{S}) \preceq^E \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$.

According to the definitions in Section 4.1, we have three resolution adequacy properties, related by the usual order of implication:

$$\mathcal{RA}_{\preceq^E_S}(\mathcal{S}) \Rightarrow \mathcal{RA}_{\preceq^E_W}(\mathcal{S}) \Rightarrow \mathcal{RA}_{\preceq^E_\cap}(\mathcal{S}) \tag{14}$$

5. A review of extension-based argumentation semantics

To make the paper self-contained, in this section we review the definition of several argumentation semantics which will be evaluated against the criteria defined in previous sections.

5.1. Traditional semantics

Stable semantics, one of the earliest multiple-status approaches and still largely adopted in several application domains, relies on the idea that an extension should not only be internally consistent but also able to reject the arguments that are outside the extension. This reasoning leads to the notion of stable extension [14,23].

Definition 31. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $E \subseteq \mathcal{A}$ is a *stable extension* of AF if and only if E is conflict-free and $\forall \alpha \in \mathcal{A}$: $\alpha \notin E$, $E \rightarrow \alpha$.

Stable semantics will be denoted as $\mathcal{ST}$, and, accordingly, the set of all the stable extensions of AF will be denoted as $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$.

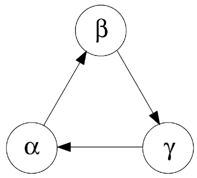

Fig. 1. A three-length cycle.

Stable semantics suffers from a significant limitation since there are argumentation frameworks, like the one shown in Fig. 1, where no extension complying with Definition 31 exists. No other semantics considered in this paper is affected by this problem except the prudent version of stable semantics.

The requirement that an extension should attack all other external arguments can be relaxed by imposing that an extension is simply able to defend itself from external attacks. This is at the basis of the notions of acceptable argument and admissible set [14], already recalled in Section 3.2. The set of the arguments acceptable with respect to a set E is traditionally denoted using the characteristic function $\mathrm{F}_{\mathrm{AF}}(E)$.

Definition 32. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, the function $\mathrm{F}_{\mathrm{AF}}: 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ which, given a set $E \subseteq \mathcal{A}$, returns the set of the acceptable arguments with respect to E, is called the *characteristic function* of AF.

Building on these concepts, the notion of complete extension can be introduced, which plays a key role in Dung's theory, since all semantics encompassed by his framework select their extensions among the complete ones.

Definition 33. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $E \subseteq \mathcal{A}$ is a *complete extension* if and only if E is admissible and every argument of $\mathcal{A}$ which is acceptable with respect to E belongs to E, i.e. $E \in \mathcal{AS}(\mathrm{AF}) \wedge \forall \alpha \in \mathrm{F}_{\mathrm{AF}}(E)\ \alpha \in E$.

The notion of complete extension does not entail the property of I-maximality. In fact, Definition 33 only establishes that nodes already defended by an extension E are included in E, but does not impose that nodes (or sets of nodes) that defend themselves are added to the extension. Probably for this reason, the notion of complete extension is not associated to a notion of *complete semantics* in [14], but rather represents an intermediate step towards the definition of grounded and preferred semantics. However, the term complete semantics has subsequently gained acceptance in the literature (see for instance [9,18]) and will be used in the present analysis to refer to the properties of the set of complete extensions. Complete semantics will be denoted as $\mathcal{CO}$.

The well-known grounded semantics, denoted as $\mathcal{GR}$, belongs to the unique-status approach and its unique extension, denoted as GE(AF), can be defined as the least fixed point of the characteristic function.

Definition 34. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, the *grounded extension* of AF, denoted as GE(AF), is the least (with respect to set inclusion) fixed point of F_{AF}.

Preferred semantics, denoted as $\mathcal{PR}$, is obtained by simply requiring the property of maximality along with admissibility.

Definition 35. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $E \subseteq \mathcal{A}$ is a *preferred extension* of AF if and only if it is a maximal (with respect to set inclusion) admissible set, i.e. a maximal element of $\mathcal{AS}(\mathrm{AF})$.

5.2. SCC-recursiveness and CF2 semantics

In [6] a general schema for argumentation semantics, called *SCC-recursiveness*, has been introduced. SCC-recursiveness is a property of the extensions which relies on the graph theoretical notion of *strongly connected components* (SCCs).

Definition 36. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, the binary relation of path-equivalence between nodes, denoted as $PE_{\mathrm{AF}} \subseteq (\mathcal{A} \times \mathcal{A})$, is defined as follows:

- $\forall \alpha \in \mathcal{A}$, $(\alpha, \alpha) \in PE_{\mathrm{AF}}$;
- given two distinct nodes $\alpha, \beta \in \mathcal{A}$, $(\alpha, \beta) \in PE_{\mathrm{AF}}$ if and only if there is a (directed) path from α to β and a (directed) path from β to α.

The *strongly connected components* of AF are the equivalence classes of nodes under the relation of path-equivalence. The set of the SCCs of AF is denoted as $\mathrm{SCCS}_{\mathrm{AF}}$.

A particular case is represented by the empty argumentation framework: when $\mathrm{AF} = \langle \emptyset, \emptyset \rangle$ we assume $\mathrm{SCCS}_{\mathrm{AF}} = \{\emptyset\}$.

We extend to SCCs the notion of parents, namely the set of the other SCCs that attack a SCC S, which is denoted as $\mathrm{sccpar}_{\mathrm{AF}}(S)$, and we introduce the definition of *proper ancestors*, denoted as $\mathrm{sccanc}_{\mathrm{AF}}(S)$.

Definition 37. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and a SCC $S \in \mathrm{SCCS}_{\mathrm{AF}}$, we define $\mathrm{sccpar}_{\mathrm{AF}}(S) = \{P \in \mathrm{SCCS}_{\mathrm{AF}} \mid P \neq S \text{ and } P \rightarrow S\}$ and $\mathrm{sccanc}_{\mathrm{AF}}(S) = \mathrm{sccpar}_{\mathrm{AF}}(S) \cup (\bigcup_{P \in \mathrm{sccpar}_{\mathrm{AF}}(S)} \mathrm{sccanc}_{\mathrm{AF}}(P))$. A SCC S such that $\mathrm{sccpar}_{\mathrm{AF}}(S) = \emptyset$ is called *initial*.

It is well-known that the graph obtained by considering SCCs as single nodes is acyclic. In other words, SCCs can be partially ordered according to the relation of attack and initial SCCs are those which are not preceded by any other one in this partial order. Of course, in any argumentation framework there is at least one initial SCC. This fact lies at the heart of the definition of SCC-recursiveness, which is based on the intuition that extensions can be built incrementally starting from initial SCCs and following the above mentioned partial order. In other words, the choices concerning extension construction carried out in an initial SCC do not depend on those concerning the other ones, while they directly affect the choices about the subsequent SCCs and so on. While the basic underlying intuition is rather simple, the formalization of SCC-recursiveness is admittedly rather complex and involves some additional notions. To keep the size of the paper within reasonable limits, we can only quickly recall the relevant definitions here, while referring the reader to [6] for more details and examples.

First of all, the choices (represented in the following definition by the set E) within the proper ancestor SCCs determine a partition of the nodes of a generic SCC S into three subsets.

Definition 38. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, a set $E \subseteq \mathcal{A}$ and a strongly connected component $S \in \mathrm{SCCS}_{\mathrm{AF}}$, we define:

- $D_{\mathrm{AF}}(S, E) = \{\alpha \in S \mid (E \cap \mathrm{outpar}_{\mathrm{AF}}(S)) \rightarrow \alpha\}$;
- $P_{\mathrm{AF}}(S, E) = \{\alpha \in S \mid (E \cap \mathrm{outpar}_{\mathrm{AF}}(S)) \not\rightarrow \alpha \wedge \exists \beta \in (\mathrm{outpar}_{\mathrm{AF}}(S) \cap \mathrm{par}_{\mathrm{AF}}(\alpha))\colon E \not\rightarrow \beta\}$;
- $U_{\mathrm{AF}}(S, E) = S \setminus (D_{\mathrm{AF}}(S, E) \cup P_{\mathrm{AF}}(S, E)) = \{\alpha \in S \mid (E \cap \mathrm{outpar}_{\mathrm{AF}}(S)) \not\rightarrow \alpha \wedge \forall \beta \in (\mathrm{outpar}_{\mathrm{AF}}(S) \cap \mathrm{par}_{\mathrm{AF}}(\alpha))\ E \rightarrow \beta\}$.

Regarding E as a part of an extension which is being constructed, the idea is that arguments in $D_{\mathrm{AF}}(S, E)$, being attacked by nodes in E, cannot be chosen in the construction of the extension E (i.e. do not belong to $E \cap S$). Selection of arguments to be included in E is therefore restricted to $(S \setminus D_{\mathrm{AF}}(S, E)) = (U_{\mathrm{AF}}(S, E) \cup P_{\mathrm{AF}}(S, E))$, which, for ease of notation, will be denoted in the following as $UP_{\mathrm{AF}}(S, E)$. Then, inspired by the reinstatement principle, we require the selection of nodes within a SCC S to be carried out on the restricted argumentation framework $\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}$ without taking into account the attacks coming from $D_{\mathrm{AF}}(S, E)$. Combining these ideas and skipping some details not strictly necessary in the context of the present paper, we can finally recall the definition of *SCC-recursiveness*.

Definition 39. A given argumentation semantics $\mathcal{S}$ is *SCC-recursive* if and only if for any argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \mathcal{GF}(\mathrm{AF}, \mathcal{A})$, where for any $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and for any set $C \subseteq \mathcal{A}$, the function $\mathcal{GF}(\mathrm{AF}, C) \subseteq 2^{\mathcal{A}}$ is defined as follows: for any $E \subseteq \mathcal{A}$, $E \in \mathcal{GF}(\mathrm{AF}, C)$ if and only if

- in case $|\,\mathrm{SCCS_{AF}}\,| = 1$, $E \in \mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$,
- otherwise, $\forall S \in \mathrm{SCCS_{AF}}\ (E \cap S) \in \mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S, E) \cap C)$,

where $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$ is a function, called *base function*, that, given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ such that $|\,\mathrm{SCCS_{AF}}\,| = 1$ and a set $C \subseteq \mathcal{A}$, gives a subset of $2^{\mathcal{A}}$.

All SCC-recursive semantics "share" this general scheme and only differ by the specific base function adopted. It has been shown in [6] that all semantics encompassed by Dung's framework are SCC-recursive and the relevant base functions have been identified. Moreover, defining and experimenting new SCC-recursive semantics is quite easy since it simply amounts to defining a base function operating on single-SCC argumentation frameworks. As shown in [6], the base function has only to respect two very simple conditions in order to ensure that the resulting extensions satisfy the requirements of being conflict-free and of being a superset of the grounded extension.

Four original SCC-recursive semantics have been defined in [6]. In particular, the SCC-recursive semantics called *CF*2 [2,6] can be regarded as the most promising, since it has been shown to provide an interesting behavior in several critical examples. Its definition relies on a very simple base function: $\mathcal{BF}_{CF2}(\mathrm{AF}, C) = \mathcal{MCF}_{\mathrm{AF}}$, where $\mathcal{MCF}_{\mathrm{AF}}$ denotes the set made up of all the maximal conflict-free sets of AF (note that the parameter C plays no role at all in this case). We recall here an important result to be used in the following.

Lemma 40. *(See Lemma 2 of [3].) For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ $\mathcal{E}_{CF2}(\mathrm{AF}) \subseteq \mathcal{MCF}_{\mathrm{AF}}$.

In words, for a generic argumentation framework AF any extension prescribed by *CF*2 semantics is a maximal conflict-free set of AF (but not vice versa).

5.3. Semi-stable semantics

Semi-stable semantics [9], denoted in the following as $\mathcal{SST}$, aims at guaranteeing the existence of extensions in any case (differently from stable semantics) while coinciding with stable semantics (differently from preferred semantics) when stable extensions exist. The definition of extensions satisfying these desiderata is ingeniously simple.

Definition 41. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ a set $E \subseteq \mathcal{A}$ is a semi-stable extension if and only if E is a complete extension such that $E \cup \mathrm{outchild_{AF}}(E)$ is maximal with respect to set inclusion.

5.4. Ideal semantics

Ideal semantics [15] provides an alternative unique-status approach which is less skeptical than grounded semantics, i.e. for any argumentation framework the (unique) ideal extension is a (sometimes strict) superset of the grounded extension. Also in this case the definition is quite simple.

Definition 42. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ a set $E \subseteq \mathcal{A}$ is ideal if and only if E is admissible and $\forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ E \subseteq P$. The ideal extension is the maximal (with respect to set inclusion) ideal set.

We will use the symbol $\mathcal{ID}$ to refer to the ideal semantics, and the ideal extension of an argumentation framework AF will be denoted as *ID*(AF).

5.5. Prudent semantics

Prudent semantics [12,13] emphasizes the role of indirect attacks.

Definition 43. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, an argument α indirectly attacks another argument β, denoted as $\alpha \hookrightarrow \beta$, if there is an odd-length path from α to β in the defeat graph corresponding to AF. A set S is without indirect conflicts, denoted as $icf(S)$, if and only if $\nexists \alpha, \beta \in S$: $\alpha \hookrightarrow \beta$.

Forbidding indirect attacks leads to the definition of p(rudent)-admissible sets.

Definition 44. Given an argumentation framework $AF = \langle \mathcal{A}, \rightarrow \rangle$, a set of arguments $S \subseteq \mathcal{A}$ is p(rudent)-admissible if and only if $\forall \alpha \in S$ α is acceptable with respect to S and $icf(S)$.

On this basis, the prudent version of several traditional notions of extensions (and of the relevant semantics) has been defined.

Definition 45. Given an argumentation framework $AF = \langle \mathcal{A}, \rightarrow \rangle$, a set of arguments $S \subseteq \mathcal{A}$ is:

- a preferred p-extension if and only if S is a maximal (with respect to set inclusion) p-admissible set;
- a stable p-extension if and only if $icf(S)$ and $\forall \alpha \in (\mathcal{A} \setminus S)$ $S \rightarrow \alpha$;
- a complete p-extension if and only if S is p-admissible and there is no argument $\alpha \notin S$ such that α is acceptable with respect to S and $icf(S \cup \{\alpha\})$.

As to the prudent version of grounded semantics, first the notion of p(rudent)-characteristic function is needed.

Definition 46. Given an argumentation framework $AF = \langle \mathcal{A}, \rightarrow \rangle$, $F^p_{AF} : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$, called *p-characteristic function* of AF, is defined as follows: given a set $S \subseteq \mathcal{A}$, $F^p_{AF}(S) = \{\alpha \in \mathcal{A} \mid \alpha$ is acceptable with respect to S and $icf(S \cup \{\alpha\})\}$.

In general F^p_{AF} is not monotonic with respect to set inclusion, one can rely however on the fact that the sequence $F^{p,i}_{AF}(\emptyset)$, $i \in \mathbb{N}$, is monotonic, where $F^{p,1}_{AF}(S) = F^p_{AF}(S)$, and $F^{p,i}_{AF}(S) = F^p_{AF}(F^{p,i-1}_{AF}(S))$, $i > 1$. Accordingly, the following definition is given in [12].

Definition 47. Given an argumentation framework $AF = \langle \mathcal{A}, \rightarrow \rangle$, let j be the lowest integer such that the sequence $F^{p,i}_{AF}(\emptyset)$ is stationary for $i \geqslant j$. Then $F^{p,j}_{AF}(\emptyset)$ is the grounded p-extension of AF, denoted as GPE(AF).

The prudent versions of grounded, complete, preferred and stable semantics will be denoted as $\mathcal{GRP}$, $\mathcal{COP}$, $\mathcal{PRP}$ and $\mathcal{STP}$, respectively.

6. Evaluating semantics

In this section we discuss the application of the evaluation criteria presented in Section 3 and of the adequacy criteria introduced in Sections 4.3 and 4.4 to the semantics reviewed in Section 5. A synthetic view of the results is given in Table 1 (for criteria defined in Section 3) and Table 2 (for adequacy criteria).

6.1. I-*maximality*

As mentioned in Section 3.1, the I-maximality criterion is necessarily satisfied by any unique-status semantics, therefore in particular by grounded, ideal and grounded prudent semantics. The definition of preferred and preferred

Table 1
Satisfaction of non-skepticism-related criteria by argumentation semantics

	$\mathcal{GR}$	$\mathcal{CO}$	$\mathcal{ST}$	$\mathcal{PR}$	$\mathcal{CF}2$	$\mathcal{SST}$	$\mathcal{ID}$	$\mathcal{GRP}$	$\mathcal{COP}$	$\mathcal{STP}$	$\mathcal{PRP}$
I-maximality (Def. 9)	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes
Admissibility (Def. 12)	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Strong adm. (Def. 14)	Yes	No	No	No	No	No	No	Yes	No	No	No
Reinstatement (Def. 15)	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	Yes	No
Weak reinst. (Def. 16)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No
$\mathcal{CF}$-reinst. (Def. 17)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No
Directionality (Def. 19)	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	No	No

Table 2
Satisfaction of adequacy criteria by argumentation semantics

	$\mathcal{GR}$	$\mathcal{CO}$	$\mathcal{ST}$	$\mathcal{PR}$	$\mathcal{CF}2$	$\mathcal{SST}$	$\mathcal{ID}$	$\mathcal{GRP}$	$\mathcal{COP}$	$\mathcal{STP}$	$\mathcal{PRP}$
$\preceq^E_\cap$-sk. ad. (Defs. 21&28)	Yes	Yes	Yes	No	Yes	No	No	No	No	No	No
$\preceq^E_W$-sk. ad. (Defs. 22&28)	Yes	Yes	Yes	No	Yes	No	No	No	No	No	No
$\preceq^E_S$-sk. ad. (Defs. 23&28)	Yes	No	No	No	No	No	No	No	No	No	No
$\preceq^E_\cap$-res. ad. (Defs. 21&30)	No	No	Yes	Yes	No	Yes	No	Yes	No	Yes	No
$\preceq^E_W$-res. ad. (Defs. 22&30)	No	No	Yes	Yes	No	Yes	No	No	No	Yes	No
$\preceq^E_S$-res. ad. (Defs. 23&30)	No	No	Yes	Yes	No	No	No	No	No	No	No

prudent extensions explicitly requires maximality of sets, which obviously entails I-maximality. This extends to semi-stable semantics too, as it is proved in [9] that every semi-stable extension is also a preferred extension. Any stable (or stable prudent) extension E attacks any argument $\alpha \notin E$, therefore any strict superset of E is not conflict-free, again entailing I-maximality. As to $CF2$ semantics, Lemma 40 directly implies I-maximality. Finally, as already mentioned in Section 5.1, it is easy to see that the set of complete (and analogously complete prudent) extensions does not satisfy I-maximality in general.

6.2. Admissibility and reinstatement

Any complete extension satisfies, by Definition 33, the properties of admissibility (1) and reinstatement (3) (which in turn entails weak (4) and $\mathcal{CF}$ (5) reinstatement). As a consequence, any semantics whose extensions are complete extensions (grounded, preferred, stable, semi-stable, stable prudent) satisfies these properties as well.

The ideal extension satisfies admissibility directly by Definition 42. We prove now that it satisfies reinstatement (and therefore also its weaker versions).

Proposition 48. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*, for any argument* α *such that* $\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)$ $ID(\mathrm{AF}) \rightarrow \beta$*, it holds that* $\alpha \in ID(\mathrm{AF})$.

Proof. Suppose by contradiction that $\exists \alpha \notin ID(\mathrm{AF})$ such that $\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha), ID(\mathrm{AF}) \rightarrow \beta$. Note that since $ID(\mathrm{AF})$ is admissible and attacks all the defeaters of α by Dung's fundamental lemma [14] it must be the case that $ID(\mathrm{AF}) \cup \{\alpha\}$ is admissible. Then the absurd hypothesis may only hold if α is not included in the intersection of the preferred extensions of AF, namely $\exists E \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$: $\alpha \notin E$. However, $\forall E \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$ $E \cup \{\alpha\}$ is admissible by Dung's fundamental lemma, since E is admissible and $ID(\mathrm{AF}) \subseteq E$, thus $\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)$ $E \rightarrow \beta$. This contradicts the fact that E, being a preferred extension, is a maximal admissible set. □

While the semantics examined so far satisfy admissibility and all forms of reinstatement at the same time, this is not the case for the remaining semantics. Let us consider admissibility first. Preferred prudent and complete prudent semantics satisfy admissibility (which is implied by p-admissibility) by Definition 45. The grounded prudent semantics satisfies it too, since it also satisfies strong admissibility, as it will be seen later. On the other hand, $CF2$ semantics does not satisfy admissibility, as it can be seen by considering the argumentation framework shown in Fig. 1, which consists of a single strongly connected component. Then $\mathcal{E}_{CF2}(\mathrm{AF}) = \mathcal{BF}_{CF2}(\mathrm{AF}, \mathcal{A}) = \mathcal{MCF}_{\mathrm{AF}} = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$: clearly, none of these extensions is an admissible set.

Turning to reinstatement, a single example, taken from [13], rules it out (even in weaker forms) for the prudent versions of complete, grounded and preferred semantics. Considering Fig. 2, we have that:

- $\{\alpha, \epsilon\}$ is a preferred p-extension as it is maximal p-admissible (δ cannot be included as it indirectly conflicts with α);
- $\{\alpha, \epsilon\}$ is also a complete p-extension for the same reason;
- $\{\delta, \epsilon\}$ is the grounded p-extension, in fact $\mathrm{F}^p_{\mathrm{AF}}(\emptyset) = \{\delta, \epsilon\} = \mathrm{F}^{p,i}_{\mathrm{AF}}(\emptyset)$ for any i since, in particular, α cannot be added as it indirectly conflicts with δ.

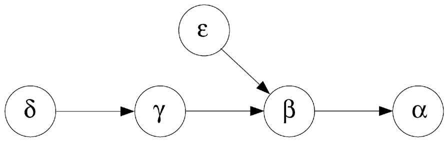

Fig. 2. Direct and indirect attacks.

Then it is evident that an initial argument (in this case δ) is not included in a preferred (and complete) prudent extension: this is incompatible with any form of reinstatement. The grounded prudent extension necessarily includes the initial nodes, but in this example it excludes the argument α which is strongly defended by the extension itself and is not in (direct) conflict with it. Therefore also in this case no form of reinstatement is satisfied.

Turning to *CF*2 semantics, the already mentioned example of Fig. 1 also shows that reinstatement is violated. Considering for instance the extension $\{\alpha\}$ we have that all the defeaters of γ (actually β) are attacked by the extension, however γ does not belong to the extension. On the other hand, *CF*2 semantics satisfies both weaker forms of reinstatement. As to $\mathcal{CF}$-reinstatement, by Lemma 40 it is easy to see that *CF*2 semantics satisfies condition (5) since any extension satisfies the stronger condition $cf(E \cup \{\alpha\}) \Rightarrow \alpha \in E$. As to weak reinstatement, it is known [6] that $\forall E \in \mathcal{E}_{CF2}(\mathrm{AF})\ \mathrm{GE}(\mathrm{AF}) \subseteq E$: then (4) follows from Proposition 50(b).

To conclude this section, let us turn to strong admissibility, i.e. condition (2). It is easy to see that this condition holds for grounded semantics.

Proposition 49. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\forall \alpha \in \mathrm{GE}(\mathrm{AF})\ sd(\alpha, \mathrm{GE}(\mathrm{AF}))$.

Proof. It is known [14] that, for any finite AF, $\mathrm{GE}(\mathrm{AF}) = \bigcup_{i \geqslant 1} \mathrm{F}^i_{\mathrm{AF}}(\emptyset)$, where $\mathrm{F}^1_{\mathrm{AF}}(\emptyset) = \mathrm{F}_{\mathrm{AF}}(\emptyset)$ and $\mathrm{F}^i_{\mathrm{AF}}(\emptyset) = \mathrm{F}_{\mathrm{AF}}(\mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$. We prove the claim by induction on the sets $\mathrm{F}^i_{\mathrm{AF}}(S)$. First, any argument α belonging to $\mathrm{F}^1_{\mathrm{AF}}(\emptyset) = \mathrm{F}_{\mathrm{AF}}(\emptyset)$ (actually, the initial arguments of the defeat graph) is strongly defended by any set, thus in particular by $\mathrm{F}^1_{\mathrm{AF}}(\emptyset)$ and GE(AF). Now, assume inductively that $\forall \alpha \in \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset)\ sd(\alpha, \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ (and therefore $\forall \alpha \in \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset)\ sd(\alpha, \mathrm{GE}(\mathrm{AF}))$). Then $\forall \beta \in (\mathrm{F}^i_{\mathrm{AF}}(\emptyset) \setminus \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ β is acceptable with respect to $\mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset)$, and since $\forall \alpha \in \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset)\ sd(\alpha, \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ then $sd(\beta, \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ holds, entailing $sd(\beta, \mathrm{F}^i_{\mathrm{AF}}(\emptyset))$ and $sd(\beta, \mathrm{GE}(\mathrm{AF}))$. □

It is also interesting to note that this notion, in combination with the dual property of weak reinstatement (4) can be used for a defense-based characterization of the grounded semantics, which, as to our knowledge, has not been previously provided in the literature. First, Proposition 50 shows that, for a given semantics $\mathcal{S}$, satisfying condition (4) is equivalent to the property of *agreement* with grounded semantics, i.e. for any argumentation framework AF and $\forall E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})\ \mathrm{GE}(\mathrm{AF}) \subseteq E$.

Proposition 50. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $E \subseteq \mathcal{A}$, E *satisfies condition* (4) *if and only if* $\mathrm{GE}(\mathrm{AF}) \subseteq E$.

Proof. (a) (4) $\Rightarrow \mathrm{GE}(\mathrm{AF}) \subseteq E$.

By the proof of Proposition 49, we have that $\mathrm{GE}(\mathrm{AF}) = \bigcup_{i \geqslant 1} \mathrm{F}^i_{\mathrm{AF}}(\emptyset)$, and $\forall i, \forall \beta \in (\mathrm{F}^i_{\mathrm{AF}}(\emptyset) \setminus \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ $sd(\beta, \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$. First, any argument $\alpha \in \mathrm{F}^1_{\mathrm{AF}}(\emptyset)$ is initial, thus it is strongly defended by any set and necessarily belongs to E according to condition (4). Now, assume inductively that $\mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset) \subseteq E$. Since $\forall \beta \in (\mathrm{F}^i_{\mathrm{AF}}(\emptyset) \setminus \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ $sd(\beta, \mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset))$ and $\mathrm{F}^{i-1}_{\mathrm{AF}}(\emptyset) \subseteq E$, we have that $sd(\beta, E)$, which by (4) implies $\beta \in E$. Then the conclusion follows.

(b) $\mathrm{GE}(\mathrm{AF}) \subseteq E \Rightarrow$ (4).

We prove that assuming $\exists \alpha \notin \mathrm{GE}(\mathrm{AF})$: $sd(\alpha, E)$ yields a contradiction, which entails the desired conclusion since $\mathrm{GE}(\mathrm{AF}) \subseteq E$. By assumption, $\forall \beta \in \mathrm{par}_{\mathrm{AF}}(\alpha)$, $\exists \gamma \in E \setminus \{\alpha\}$: $\gamma \rightarrow \beta \wedge sd(\gamma, E \setminus \{\alpha\})$. Since GE(AF) is a complete extension, it must be the case that one of such γ exists with $\gamma \notin \mathrm{GE}(\mathrm{AF})$, otherwise $\alpha \in \mathrm{GE}(\mathrm{AF})$. Thus, in particular $\exists \gamma \notin \mathrm{GE}(\mathrm{AF})$: $sd(\gamma, E \setminus \{\alpha\})$, where $\mathrm{GE}(\mathrm{AF}) \subseteq (E \setminus \{\alpha\})$. Now, iterating the same kind of reasoning on γ we are led to consider an argument $\gamma' \in E \setminus \{\alpha, \gamma\}$: $sd(\gamma', E \setminus \{\alpha, \gamma\}) \wedge \gamma' \notin \mathrm{GE}(\mathrm{AF})$. This in turn leads to consider an argument $\gamma'' \in E \setminus \{\alpha, \gamma, \gamma'\}$: $sd(\gamma'', E \setminus \{\alpha, \gamma, \gamma'\}) \wedge \gamma'' \notin \mathrm{GE}(\mathrm{AF})$, and so on. In summary we are led to consider an infinite sequence of distinct arguments within E, which is impossible due to the finiteness of $\mathcal{A}$. □

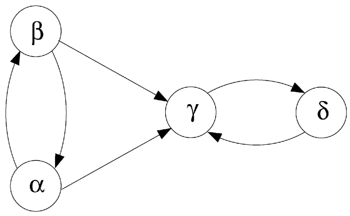

Fig. 3. A case where $ID(\mathrm{AF}) \supsetneq \mathrm{GE}(\mathrm{AF})$.

On the other hand, condition (2) implies that an extension E is included in the grounded extension.

Proposition 51. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $E \subseteq \mathcal{A}$*, if* E *satisfies condition* (2) *then* $E \subseteq \mathrm{GE}(\mathrm{AF})$.

Proof. We prove that assuming $\exists \alpha \notin \mathrm{GE}(\mathrm{AF})$: $sd(\alpha, E)$ yields a contradiction, which by condition (2) entails the conclusion $E \subseteq \mathrm{GE}(\mathrm{AF})$. Note first that, by assumption, $\exists \beta \in \mathcal{A}$: $\beta \rightarrow \alpha$, since initial nodes belong to GE(AF). Then, according to (2), for any such β there must be an argument $\gamma \in E \setminus \{\alpha\}$: $\gamma \rightarrow \beta \wedge sd(\gamma, E \setminus \{\alpha\})$. As in the proof of Proposition 50(b), it must be the case that for at least one of these γ, $\gamma \notin \mathrm{GE}(\mathrm{AF})$, and we are led to consider an infinite sequence of distinct arguments within E, which is impossible. □

Propositions above show that the conjunction of weak reinstatement and strong admissibility provides a characterization of grounded semantics. In particular, any semantics whose extensions are not included in the grounded extension can not satisfy strong admissibility. For instance, considering the simple example of mutual attack (often called Nixon diamond) corresponding to the argumentation framework $\mathrm{AF} = \langle \{\alpha, \beta\}, \{(\alpha, \beta), (\beta, \alpha)\} \rangle$, any multiple-status semantics $\mathcal{S}$ such that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) \supseteq \{\{\alpha\}, \{\beta\}\}$ violates condition (2). This immediately rules out complete, stable, preferred, *CF*2, semi-stable, complete prudent, stable prudent and preferred prudent semantics. Also ideal semantics does not satisfy (2) since there are cases where $\mathrm{GE}(\mathrm{AF}) \subsetneq ID(\mathrm{AF})$. For instance, in the example of Fig. 3 $\mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) = \{\{\alpha, \delta\}, \{\beta, \delta\}\}$, then $ID(\mathrm{AF}) = \{\delta\}$ which is admissible and included in both preferred extensions. However, $\mathrm{GE}(\mathrm{AF}) = \emptyset$.

Finally, it can be shown (proof is omitted) that the grounded prudent extension GPE(AF) satisfies strong admissibility.

Proposition 52. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\forall \alpha \in \mathrm{GPE}(\mathrm{AF})$ $sd(\alpha, \mathrm{GPE}(\mathrm{AF}))$.

6.3. Directionality

To begin our analysis, we prove that the directionality criterion is satisfied by all SCC-recursive semantics such that for any argumentation framework the existence of extensions is guaranteed. This covers the cases of grounded, complete, and preferred semantics, whose SCC-recursiveness has been proved in [6], and of *CF*2 semantics, which, as recalled in Section 5.2, is directly defined in SCC-recursive terms. Two preliminary lemmata, whose easy proof is omitted, are required.

Lemma 53. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $U \in \mathcal{US}(\mathrm{AF})$, $\forall S \in \mathrm{SCCS}_{\mathrm{AF}}$: $(S \cap U) \neq \emptyset$, $S \subseteq U$. *Then* $\exists Q \subseteq \mathrm{SCCS}_{\mathrm{AF}}$: $U = \bigcup_{S \in Q} S$.

Lemma 54. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\forall U \in \mathcal{US}(\mathrm{AF})$ $\forall S \in \mathrm{SCCS}_{\mathrm{AF}}$, $S \subseteq U \Rightarrow (\bigcup_{S' \in \mathrm{sccanc}_{\mathrm{AF}}(S)} S') \subseteq U$.

Proposition 55. *Any SCC-recursive semantics* $\mathcal{S}$ *such that* $\forall\, \mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ $\forall C \subseteq \mathcal{A}$ $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C) \neq \emptyset$, *satisfies the directionality criterion.*

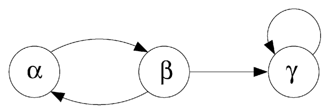

Fig. 4. Stable semantics does not satisfy directionality.

Proof. By Definition 39 it holds that for any argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$:

$$\mathcal{E}_{\mathcal{S}}(\mathrm{AF}) = \big\{E \subseteq \mathcal{A}\colon \forall S \in \mathrm{SCCS}_{\mathrm{AF}} (E \cap S) \in \mathcal{GF}\big(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S, E)\big)\big\} \tag{15}$$

Considering a generic unattacked set $U \in \mathcal{US}(\mathrm{AF})$, from Lemma 53 we know that $\exists Q \subseteq \mathrm{SCCS}_{\mathrm{AF}}\colon U = \bigcup_{S \in Q} S$ and from Lemma 54 that $\forall S \in Q$ $\mathrm{sccanc}_{\mathrm{AF}}(S) \subseteq Q$. It follows that $\mathrm{SCCS}_{\mathrm{AF}{\downarrow}_U} = Q$. Then $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_U) = \{E \subseteq U\colon \forall S \in Q\ (E \cap S) \in \mathcal{GF}((\mathrm{AF}{\downarrow}_U){\downarrow}_{UP_{\mathrm{AF}\downarrow_U}(S,E)}, U_{\mathrm{AF}\downarrow_U}(S, E))\} = \{E \subseteq U\colon \forall S \in Q\ (E \cap S) \in \mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}\downarrow_U}(S,E)}, U_{\mathrm{AF}\downarrow_U}(S, E))\}$. Now, since $\forall S \in Q$ $\mathrm{sccanc}_{\mathrm{AF}}(S) \subseteq Q$, it is easy to see that $\forall E \subseteq U, \forall S \in Q$, the following equalities hold since the restriction to U does not introduce any difference with respect to the whole AF as far as the relevant sets are concerned: $UP_{\mathrm{AF}\downarrow_U}(S, E) = UP_{\mathrm{AF}}(S, E)$ and $U_{\mathrm{AF}\downarrow_U}(S, E) = U_{\mathrm{AF}}(S, E)$. Then

$$\mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_U) = \big\{E \subseteq U\colon \forall S \in Q\ (E \cap S) \in \mathcal{GF}\big(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E)}, U_{\mathrm{AF}}(S, E)\big)\big\} \tag{16}$$

According to Definition 19, we have now to show that $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, U) = \mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_U)$, where $\mathcal{AE}_{\mathcal{S}}(\mathrm{AF}, U) = \{E \cap U\colon E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})\}$. If $E \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ then from (15) and taking into account that $\forall S \in Q$ $UP_{\mathrm{AF}}(S, E)$ and $U_{\mathrm{AF}}(S, E)$ only depend on $E \cap U$ rather than E, it is easy to derive that $\forall S \in Q\ ((E \cap U) \cap S) \in \mathcal{GF}(\mathrm{AF}{\downarrow}_{UP_{\mathrm{AF}}(S,E\cap U)}, U_{\mathrm{AF}}(S, E \cap U))$, which, using (16), yields $(E \cap U) \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_U)$. On the other hand, if $E' \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF}{\downarrow}_U)$ then by (16) it satisfies condition (15) as far as any strongly connected component $S \in Q$ is concerned. Then we have to show that $\exists E^*$ such that $(E^* \cap U) = E'$ with E^* satisfying the definition of SCC-recursiveness also for any $S' \in (\mathrm{SCCS}_{\mathrm{AF}} \setminus Q)$. But since $\forall\, \mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ $\forall C \subseteq \mathcal{A}$, $\mathcal{BF}_{\mathcal{S}}(\mathrm{AF}, C)$ is defined, it is clearly possible to construct such a set E^* (in general, more than one exist) by applying Definition 39 to the strongly connected components not included in Q following the (partial) order induced among them by the attack relation (note that by Lemma 54 no strongly connected component in $(\mathrm{SCCS}_{\mathrm{AF}} \setminus Q)$ can precede any strongly connected component in Q in this order). □

Stable semantics, while being SCC-recursive [6], does not satisfy the hypothesis of Proposition 55. In fact, it does not satisfy the directionality criterion as it can be seen considering Fig. 4, where $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}) = \{\{\beta\}\}$ and therefore $\mathcal{AE}_{\mathcal{ST}}(\mathrm{AF}, \{\alpha, \beta\}) = \{\{\beta\}\}$, while $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}{\downarrow}_{\{\alpha,\beta\}}) = \{\{\alpha\}, \{\beta\}\}$. Since in this example both semi-stable and stable prudent semantics behave as stable semantics, neither of them satisfies directionality.

Recalling the argumentation framework of Fig. 2 and the fact that $\{\alpha, \epsilon\}$ is both a preferred p- and complete p-extension where the initial argument δ is not included, we immediately see that directionality is satisfied neither by preferred prudent nor by complete prudent semantics. As to grounded prudent semantics, it is easy to see that it satisfies directionality as, according to Definition 47, the grounded p-extension can be conceived as the result of a constructive process starting from initial arguments, where inclusion of an argument only depends on its ancestors.

Finally, Proposition 58, which requires two preliminary lemmata whose proof is omitted, shows that the ideal semantics satisfies the directionality criterion.

Lemma 56. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\forall U \in \mathcal{US}(\mathrm{AF})$ *it holds that*

- *for any* $E \subseteq \mathcal{A}$ *such that* $E \in \mathcal{AS}(\mathrm{AF})$, $E \cap U \in \mathcal{AS}(\mathrm{AF}{\downarrow}_U)$;
- *for any* $E \subseteq U$ *such that* $E \in \mathcal{AS}(\mathrm{AF}{\downarrow}_U)$, $E \in \mathcal{AS}(\mathrm{AF})$.

Lemma 57. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $ID(\mathrm{AF}) = \bigcup_{E \in \mathcal{AS}(\mathrm{AF})\colon \forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}) E \subseteq P} E$.

Proposition 58. *Ideal semantics satisfies the directionality criterion.*

Proof. We have to prove that for any argumentation framework AF and for any $U \in \mathcal{US}(\mathrm{AF})$, $ID(\mathrm{AF}) \cap U = ID(\mathrm{AF}{\downarrow_U})$. On the basis of Lemma 57, we have to show that

$$\left(\bigcup_{E \in \mathcal{AS}(\mathrm{AF}):\ \forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ E \subseteq P} E\right) \cap U = \bigcup_{E \in \mathcal{AS}(\mathrm{AF}{\downarrow_U}):\ \forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}{\downarrow_U})\ E \subseteq P} E$$

which is equivalent to

$$\left(\bigcup_{E \in \mathcal{AS}(\mathrm{AF}):\ \forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ E \subseteq P} E \cap U\right) = \bigcup_{E \in \mathcal{AS}(\mathrm{AF}{\downarrow_U}):\ \forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}{\downarrow_U})\ E \subseteq P} E$$

First, let us show that the set on the left-hand side of the last equality is a subset of the set on the right-hand side. By Lemma 56, if $E \in \mathcal{AS}(\mathrm{AF})$ then $(E \cap U) \in \mathcal{AS}(\mathrm{AF}{\downarrow_U})$. Moreover, $\forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$ $(E \cap U) \subseteq (P \cap U)$. Since preferred semantics satisfies directionality we have that $\{P \cap U \mid P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\} = \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}{\downarrow_U})$, from which it follows that $\forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}{\downarrow_U})$ $(E \cap U) \subseteq P$. To see that, vice versa, the set on the right-hand side of the equality is a subset of the set on the left-hand side, recall that by Lemma 56 if $E \in \mathcal{AS}(\mathrm{AF}{\downarrow_U})$ then E is also admissible in AF. Exploiting again the directionality of preferred semantics it is finally easy to see that $\forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF}{\downarrow_U})$ $E \subseteq P$ implies also that $\forall P \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$ $E \subseteq P$. □

6.4. Skepticism adequacy

In Section 4.3 three skepticism adequacy properties, ordered by implication, have been defined. First let us show that grounded semantics satisfies all of them (note that we refer to the equivalent condition stated in Proposition 29, as we will do also in some of the following proofs).

Lemma 59. *Let us consider an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *with two arguments* $\alpha, \beta \in \mathcal{A}$ *such that* $\alpha \rightarrow \beta$. *For any two sets of arguments* A^* *and* A *such that* $A^* \subseteq A$ *and* A *is admissible in* AF, $\mathrm{F}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(A^*) \subseteq \mathrm{F}_{\mathrm{AF}}(A)$.

Proof. Considering a generic $\gamma \in \mathrm{F}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(A^*)$, we have to prove that $\gamma \in \mathrm{F}_{\mathrm{AF}}(A)$, i.e. that γ is acceptable with respect to A in AF. To this purpose, let us consider a generic argument $\delta \in \mathrm{par}_{\mathrm{AF}}(\gamma)$, and let us prove that $A \rightarrow \delta$ in AF. By definition of $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}$, it is easy to see that $\delta \in \mathrm{par}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(\gamma)$, and since $\gamma \in \mathrm{F}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(A^*)$ it must be the case that $A^* \rightarrow \delta$ holds in $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}$. Since $A^* \subseteq A$, $A \rightarrow \delta$ in $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}$, i.e. $\exists \epsilon \in A$: $\epsilon \rightarrow \delta$ in $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}$. Now, we have two cases to consider. If it is not the case that $\beta = \epsilon$ and $\alpha = \delta$ then, by definition of $\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}$, $\epsilon \rightarrow \delta$ also in AF and the claim is proved. Otherwise, $\beta = \epsilon$ and $\alpha = \delta$, then, by the hypothesis that $\alpha \rightarrow \beta$ in AF it follows that $\delta \rightarrow A$ in AF, which, by admissibility of A, entails that, also in this case, $A \rightarrow \delta$ in AF. □

Proposition 60. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and two arguments* $\alpha, \beta \in \mathcal{A}$ *such that* $\alpha \rightarrow \beta$, *we have that* $\mathrm{GE}(\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}) \subseteq \mathrm{GE}(\mathrm{AF})$.

Proof. Taking into account the definition of grounded extension, it is sufficient to prove that $\forall i \geqslant 1$ $\mathrm{F}^i_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(\emptyset) \subseteq \mathrm{F}^i_{\mathrm{AF}}(\emptyset)$. This can be easily proved by induction on i, taking into account Lemma 59 and the fact that $\forall i \geqslant 1$ $\mathrm{F}^i_{\mathrm{AF}}(\emptyset)$ is admissible [14]. In particular, in the basis case Lemma 59 can be applied with $A^* = A = \emptyset$ to prove that $\mathrm{F}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(\emptyset) \subseteq \mathrm{F}_{\mathrm{AF}}(\emptyset)$, while in the induction step it can be applied with $A^* = \mathrm{F}^i_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(\emptyset)$ and $A = \mathrm{F}^i_{\mathrm{AF}}(\emptyset)$, where $A^* \subseteq A$ is inductively assumed, to prove that $\mathrm{F}^{i+1}_{\mathrm{AF}^{\rightleftarrows(\alpha,\beta)}}(\emptyset) \subseteq \mathrm{F}^{i+1}_{\mathrm{AF}}(\emptyset)$. □

Since grounded semantics belongs to the unique-status approach, it is immediate to see that the result of Proposition 60 entails the satisfaction of all skepticism adequacy properties including its strongest form, based on $\preceq^E_S$. On the other hand, it is easy to see that $\preceq^E_S$-skepticism adequacy is in contrast with the behavior of most semantics in the simple case of mutually attacking arguments. In fact, letting $\mathrm{AF}_1 = \langle \{\alpha, \beta\}, \{(\alpha, \beta)\} \rangle$ and $\mathrm{AF}_2 = \mathrm{AF}_1{}^{\rightleftarrows(\alpha,\beta)} = \langle \{\alpha, \beta\}, \{(\alpha, \beta), (\beta, \alpha)\} \rangle$, any multiple-status semantics $\mathcal{S}$ such that (as usual) $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}_1) = \{\{\alpha\}\}$ and $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}_2) \supseteq \{\{\alpha\}, \{\beta\}\}$ does not satisfy $\preceq^E_S$-skepticism adequacy. This rules out all the remaining semantics except ideal and grounded prudent semantics, which however will be shown to fail to satisfy any form of skepticism adequacy.

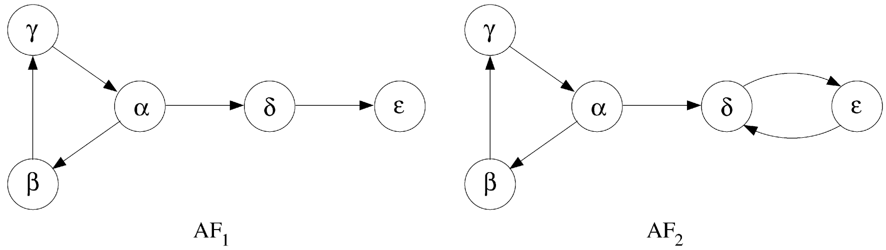

Fig. 5. Several semantics are not skepticism adequate.

$\preceq^E_W$-skepticism adequacy (along with the implied weaker form $\preceq^E_\cap$) turns out to be a more reasonable requirement, which we now show to be satisfied by complete, stable and *CF*2 semantics. Proposition 61 provides the result for complete semantics.

Proposition 61. *Given two argumentation frameworks* $\mathrm{AF}_1 = \langle \mathcal{A}, \rightarrow_1 \rangle$ *and* $\mathrm{AF}_2 = \langle \mathcal{A}, \rightarrow_2 \rangle$ *such that* $\mathrm{AF}_1 \preceq^A \mathrm{AF}_2$, $\mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_1) \preceq^E_W \mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_2)$.

Proof. We have to show that $\forall E_2 \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_2)\ \exists E_1 \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_1)$: $E_1 \subseteq E_2$. The conclusion easily follows from the fact that, as well known, for any AF $\mathrm{GE(AF)} \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})$ and $\forall E \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF})\ \mathrm{GE(AF)} \subseteq E$. Moreover, from Proposition 60, we have that $\mathrm{GE}(\mathrm{AF}_1) \subseteq \mathrm{GE}(\mathrm{AF}_2)$. Then $\forall E_2 \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_2)\ \exists E_1 = \mathrm{GE}(\mathrm{AF}_1) \in \mathcal{E}_{\mathcal{CO}}(\mathrm{AF}_1)$: $E_1 \subseteq \mathrm{GE}(\mathrm{AF}_2) \subseteq E_2$. □

Turning to stable semantics, it is easy to see that it satisfies $\preceq^E_W$-skepticism adequacy in all cases where stable extensions exist for both the argumentation frameworks considered in the comparison (otherwise, one (or both) of the terms of comparison is undefined and nothing can be said).

Proposition 62. *Given two argumentation frameworks* $\mathrm{AF}_1 = \langle \mathcal{A}, \rightarrow_1 \rangle \in \mathcal{D}_{\mathcal{ST}}$ *and* $\mathrm{AF}_2 = \langle \mathcal{A}, \rightarrow_2 \rangle \in \mathcal{D}_{\mathcal{ST}}$ *such that* $\mathrm{AF}_1 \preceq^A \mathrm{AF}_2$, $\mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_1) \preceq^E_W \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_2)$.

Proof. We have to show that $\forall E_2 \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_2)\ \exists E_1 \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_1)$: $E_1 \subseteq E_2$. It is easy to prove in particular that $\forall E_2 \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_2)\ E_2 \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}_1)$. In fact, since $\mathcal{CONF}(\mathrm{AF}_1) = \mathcal{CONF}(\mathrm{AF}_2)$ if E_2 is conflict-free in AF_2 it is also so in AF_1. Moreover, since $\rightarrow_2 \subseteq \rightarrow_1$, given that $\forall \alpha \in (\mathcal{A} \setminus E_2)\ E_2 \rightarrow_2 \alpha$, then $\forall \alpha \in (\mathcal{A} \setminus E_2)\ E_2 \rightarrow_1 \alpha$. Therefore E_2 is a stable extension of AF_1. □

Finally, also *CF*2 semantics satisfies $\preceq^E_W$-skepticism adequacy. This follows from the (actually stronger) result in Proposition 63, proved in [3].

Proposition 63. *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and two arguments* $\alpha, \beta \in \mathcal{A}$ *such that* $\alpha \neq \beta \wedge \alpha \rightarrow \beta$, $\mathcal{E}_{CF2}(\mathrm{AF}) \subseteq \mathcal{E}_{CF2}(\mathrm{AF}^{\rightleftarrows(\alpha,\beta)})$.

The remaining semantics fail to satisfy $\preceq^E_\cap$-skepticism adequacy and therefore skepticism adequacy in any form. Consider first the example in Fig. 5. It can be seen that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}_1) = \{\emptyset\}$ for preferred, ideal, semi-stable and preferred prudent semantics. However, for the same semantics it holds that $\mathcal{E}_{\mathcal{S}}(\mathrm{AF}_2) = \{\epsilon\}$. Also stable prudent semantics fails to satisfy $\preceq^E_\cap$-skepticism adequacy, as shown by the example in Fig. 6. In fact AF_1 admits exactly one stable prudent extension, namely $\{\alpha, \delta\}$, while AF_2 admits as unique stable prudent extension the set $\{\alpha, \gamma\}$. As to grounded prudent semantics, consider Fig. 7. It is easy to see that $\mathrm{GPE}(\mathrm{AF}_1) = \{\alpha, \gamma\}$ (in particular ζ cannot be included as it indirectly conflicts with α). On the other hand, considering $\mathrm{AF}_2 = \mathrm{AF}_1^{\rightleftarrows(\alpha,\beta)}$, we have $\mathrm{GPE}(\mathrm{AF}_2) = \{\gamma, \zeta\}$. As for complete prudent semantics, consider Fig. 8: it turns out that $\mathcal{E}_{\mathcal{COP}}(\mathrm{AF}_1) = \{\{\alpha, \gamma_2\}, \{\gamma_2, \gamma_5\}\}$, while $\mathcal{E}_{\mathcal{COP}}(\mathrm{AF}_2) = \{\{\alpha\}\}$.

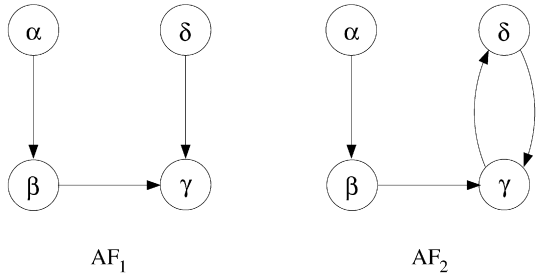

Fig. 6. Stable prudent semantics is not skepticism adequate.

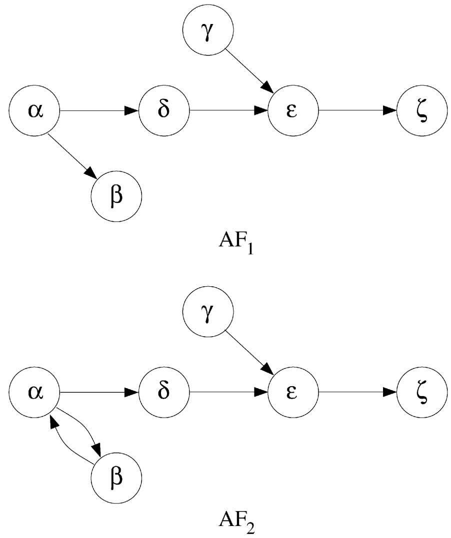

Fig. 7. Grounded prudent semantics is not skepticism adequate.

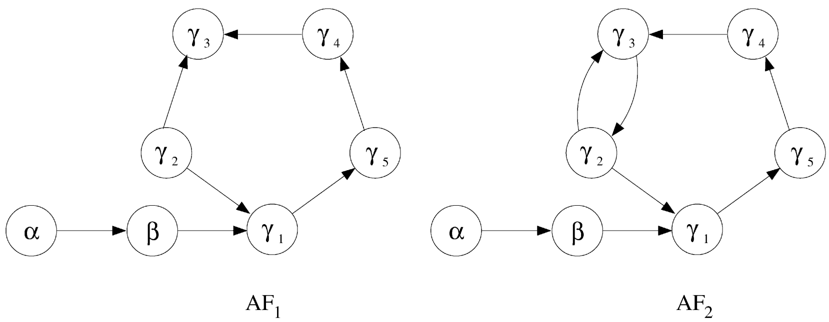

Fig. 8. Complete prudent semantics is not skepticism adequate.

6.5. Resolution adequacy

The criterion of resolution adequacy is satisfied in all forms by preferred[3] and stable semantics. The proof concerning preferred semantics requires a preliminary lemma.

Lemma 64. *(See Lemma 1 of [18].) Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $E \subseteq \mathcal{A}$, $E \in \mathcal{AS}(\mathrm{AF})$ *if and only if* $\exists\, \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$: $E \in \mathcal{AS}(\mathrm{AF}')$.

Proposition 65. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathcal{UR}(\mathrm{AF}, \mathcal{PR}) \preceq^E_S \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$.

Proof. We have to prove the conditions: (i) $\forall E_2 \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ \exists E_1 \in \mathcal{UR}(\mathrm{AF}, \mathcal{PR})$: $E_1 \subseteq E_2$; (ii) $\forall E_1 \in \mathcal{UR}(\mathrm{AF}, \mathcal{PR})\ \exists E_2 \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})$: $E_1 \subseteq E_2$.

From Lemma 64 it follows in particular that for any maximal admissible set E_2 (actually, a preferred extension) of AF, there is $\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ such that E_2 is admissible in AF′. Then E_2 is maximal admissible in AF′ (since there cannot be an admissible set of AF′ which is not an admissible set of AF), and therefore $E_2 \in \mathcal{UR}(\mathrm{AF}, \mathcal{PR})$. Thus, $\forall E_2 \in \mathcal{E}_{\mathcal{PR}}(\mathrm{AF})\ E_2 \in \mathcal{UR}(\mathrm{AF}, \mathcal{PR})$, which proves condition (i). On the other hand, since any $E_1 \in \mathcal{UR}(\mathrm{AF}, \mathcal{PR})$

[3] Some results related to resolution adequacy of grounded, complete, and preferred semantics are provided in [18] and are generalized here.

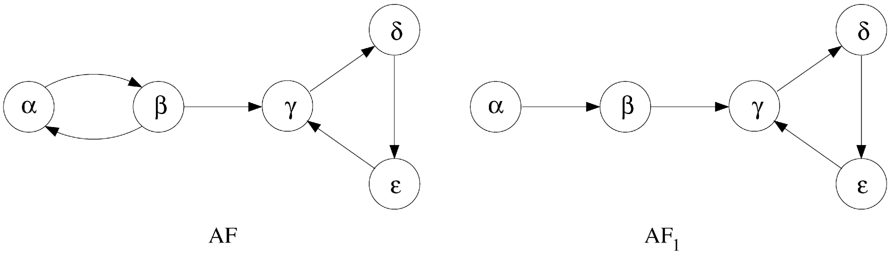

Fig. 9. Semi-stable semantics is not $\preceq_S^E$-resolution adequate.

is a (maximal) admissible set in an $\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ we know from Lemma 64 that E_1 is admissible in AF and is therefore included in one of the maximal admissible sets of AF, which is exactly the desired condition (ii). □

A similar result holds for stable extensions, provided that their existence in AF and in all elements of $\mathcal{RES}(\mathrm{AF})$ is assumed.

Proposition 66. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *such that* $\mathrm{AF} \in \mathcal{D}_{\mathcal{ST}}$ *and* $\forall\, \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})\ \mathrm{AF}' \in \mathcal{D}_{\mathcal{ST}}$, $\mathcal{UR}(\mathrm{AF}, \mathcal{ST}) \preceq_S^E \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$.

Proof. We actually prove the stronger result $\mathcal{UR}(\mathrm{AF}, \mathcal{ST}) = \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$. $\mathcal{UR}(\mathrm{AF}, \mathcal{ST}) \subseteq \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$ follows from the fact that any stable extension of any $\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ is also a stable extension of AF (see the proof of Proposition 62). On the other hand, given $S \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF})$, we have to show that there is at least an $\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ such that $S \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}')$. If no element of S is involved in a mutual attack, it is easy to see that $\forall\, \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})\ S \in \mathcal{E}_{\mathcal{ST}}(\mathrm{AF}')$. Otherwise, such an AF' is obtained from AF by suppressing, for any mutual attack involving an argument $\alpha \in S$, the edge incoming into α (clearly there are no mutual attacks within S as it is conflict-free). □

Semantics which are partially inspired by the notion of stable extension, namely semi-stable and stable prudent semantics, are unable to satisfy $\preceq_S^E$-resolution adequacy while complying with its weaker forms. As to negative results, Fig. 9 shows that $\preceq_S^E$-resolution adequacy does not hold for semi-stable semantics. In fact, $E_1 = \{\alpha\}$ is the only semi-stable extension of $\mathrm{AF}_1 \in \mathcal{RES}(\mathrm{AF})$ but there is no E_2 among the semi-stable extensions of AF such that $E_1 \subseteq E_2$, since the only semi-stable extension of AF is $\{\beta, \delta\}$. As to stable prudent semantics, we can refer back to Fig. 6. In fact, $E_1 = \{\alpha, \delta\}$ is a stable prudent extension of $\mathrm{AF}_1 \in \mathcal{RES}(\mathrm{AF}_2)$ but there is no E_2 among the stable prudent extensions of AF_2 such that $E_1 \subseteq E_2$ (since the only stable prudent extension of AF_2 is $\{\alpha, \gamma\}$).

The following proposition shows that semi-stable semantics satisfies $\preceq_W^E$-resolution adequacy.

Proposition 67. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $\mathcal{UR}(\mathrm{AF}, \mathcal{SST}) \preceq_W^E \mathcal{E}_{\mathcal{SST}}(\mathrm{AF})$.

Proof. We have to prove that $\forall E_2 \in \mathcal{E}_{\mathcal{SST}}(\mathrm{AF})\ \exists E_1 \in \mathcal{UR}(\mathrm{AF}, \mathcal{SST})$: $E_1 \subseteq E_2$. It is known that any semi-stable extension of any AF is also a preferred extension of AF [9]. Then, given a semi-stable extension $E_2 \in \mathcal{E}_{\mathcal{SST}}(\mathrm{AF})$, it can be derived from the proof of Lemma 64 given in [18] that $\exists\, \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ such that E_2 is a preferred extension of AF' and $\mathrm{outchild}_{\mathrm{AF}}(E_2) = \mathrm{outchild}_{\mathrm{AF}'}(E_2)$. Suppose by contradiction that E_2 is not also a semi-stable extension of AF', then there is a maximal admissible set S in AF' such that $(S \cup \mathrm{outchild}_{\mathrm{AF}'}(S)) \supsetneq (E_2 \cup \mathrm{outchild}_{\mathrm{AF}'}(E_2))$. Recall that $\mathrm{outchild}_{\mathrm{AF}'}(S) \subseteq \mathrm{outchild}_{\mathrm{AF}}(S)$, and that S is admissible also in AF. Then there is a maximal admissible set $S^* \in \mathrm{AF}$ such that $S^* \supseteq S$ and therefore $\mathrm{outchild}_{\mathrm{AF}}(S^*) \supseteq \mathrm{outchild}_{\mathrm{AF}}(S) \supseteq \mathrm{outchild}_{\mathrm{AF}'}(S)$. But then $(S^* \cup \mathrm{outchild}_{\mathrm{AF}}(S^*)) \supseteq (S \cup \mathrm{outchild}_{\mathrm{AF}}(S)) \supseteq (S \cup \mathrm{outchild}_{\mathrm{AF}'}(S)) \supsetneq (E_2 \cup \mathrm{outchild}_{\mathrm{AF}'}(E_2)) = (E_2 \cup \mathrm{outchild}_{\mathrm{AF}}(E_2))$, thus $(S^* \cup \mathrm{outchild}_{\mathrm{AF}}(S^*)) \supsetneq (E_2 \cup \mathrm{outchild}_{\mathrm{AF}}(E_2))$ which contradicts the fact that E_2 is a semi-stable extension of AF. □

Finally, also stable prudent semantics satisfies $\preceq_W^E$-resolution adequacy (proof is omitted).

Proposition 68. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*, such that* $\mathrm{AF} \in \mathcal{D}_{\mathcal{STP}}$ *and* $\forall\, \mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})$ $\mathrm{AF}' \in \mathcal{D}_{\mathcal{STP}}$, $\mathcal{UR}(\mathrm{AF}, \mathcal{STP}) \preceq^{E}_{W} \mathcal{E}_{\mathcal{STP}}(\mathrm{AF})$.

Turning to grounded prudent semantics, the example of Fig. 10 shows that it does not satisfy $\preceq^{E}_{W}$-resolution adequacy (and therefore neither its stronger form $\preceq^{E}_{S}$). In fact, $\mathcal{RES}(\mathrm{AF}) = \{\mathrm{AF}_1, \mathrm{AF}_2\}$, $\mathrm{GPE}(\mathrm{AF}) = \emptyset$, $\mathrm{GPE}(\mathrm{AF}_1) = \{\beta\}$, and $\mathrm{GPE}(\mathrm{AF}_2) = \{\alpha\}$, from which it is easy to see that $\preceq^{E}_{W}$-resolution adequacy is not satisfied. On the other hand grounded prudent semantics satisfies the weakest form of resolution adequacy (proof is omitted).

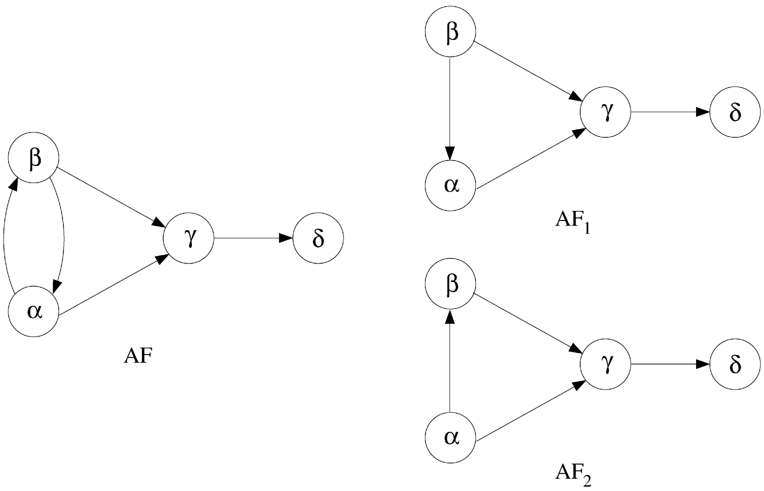

Fig. 10. Several semantics are not resolution adequate.

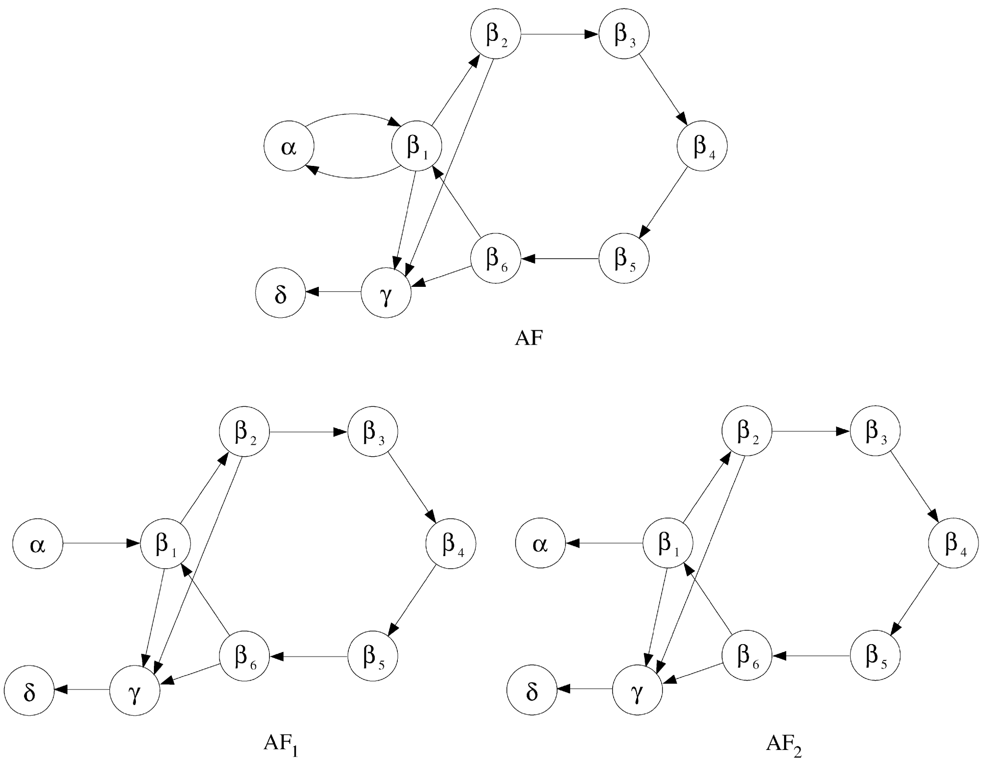

Fig. 11. *CF*2 semantics is not resolution adequate.

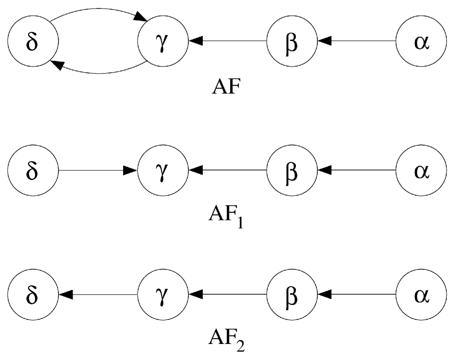

Fig. 12. Preferred prudent and complete prudent semantics are not resolution adequate.

Proposition 69. *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, $(\bigcap_{\mathrm{AF}' \in \mathcal{RES}(\mathrm{AF})} \mathrm{GPE}(\mathrm{AF}')) \subseteq \mathrm{GPE}(\mathrm{AF})$.

The remaining semantics fail to satisfy resolution adequacy in any form. To see this for grounded, ideal and complete semantics consider again Fig. 10. First, it is easy to see that $\mathcal{UR}(\mathrm{AF}, \mathcal{GR}) = \mathcal{UR}(\mathrm{AF}, \mathcal{ID}) = \mathcal{UR}(\mathrm{AF}, \mathcal{CO}) = \{\{\alpha, \delta\}, \{\beta, \delta\}\}$. Then, note that $\mathcal{E}_{\mathcal{GR}}(\mathrm{AF}) = \mathcal{E}_{\mathcal{ID}}(\mathrm{AF}) = \{\mathrm{GE}(\mathrm{AF})\} = \{ID(\mathrm{AF})\} = \{\emptyset\}$ while $\mathcal{E}_{\mathcal{CO}}(\mathrm{AF}) = \{\emptyset, \{\alpha, \delta\}, \{\beta, \delta\}\}$. As a consequence, for any $\mathcal{S} \in \{\mathcal{GR}, \mathcal{ID}, \mathcal{CO}\}$ it holds that $\mathcal{UR}(\mathrm{AF}, \mathcal{S}) \not\preceq^{E}_{\cap} \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$, since $\bigcap_{E_1 \in \mathcal{UR}(\mathrm{AF}, \mathcal{S})} E_1 = \{\delta\} \nsubseteq \emptyset = \bigcap_{E_2 \in \mathcal{E}_{\mathcal{S}}(\mathrm{AF})} E_2$. Therefore $\mathcal{UR}(\mathrm{AF}, \mathcal{S}) \not\preceq^{E} \mathcal{E}_{\mathcal{S}}(\mathrm{AF})$ for any skepticism relation $\preceq^{E}$.

As for $CF2$ semantics, consider Fig. 11 where $\mathcal{RES}(\mathrm{AF}) = \{\mathrm{AF}_1, \mathrm{AF}_2\}$. First, it is easy to see that $\{\alpha, \beta_3, \beta_5, \gamma\} \in \mathcal{E}_{CF2}(\mathrm{AF})$ therefore, in particular, $\delta \notin \bigcap_{E \in \mathcal{E}_{CF2}(\mathrm{AF})} E$. Then it can be noted that $\mathcal{E}_{CF2}(\mathrm{AF}_1) = \{\alpha, \beta_2, \beta_4, \beta_6, \delta\}$, while

$$\mathcal{E}_{CF2}(\mathrm{AF}_2) = \big\{\{\beta_1, \beta_3, \beta_5, \delta\}, \{\beta_1, \beta_4, \delta\}, \{\alpha, \beta_2, \beta_4, \beta_6, \delta\}, \{\alpha, \beta_2, \beta_5, \delta\}, \{\alpha, \beta_3, \beta_6, \delta\}\big\}$$

Thus $\delta \in \bigcap_{E \in \mathcal{UR}(\mathrm{AF}, CF2)} E$ and as a consequence $\mathcal{UR}(\mathrm{AF}, CF2) \not\preceq^{E}_{\cap} \mathcal{E}_{CF2}(\mathrm{AF})$.

Finally, the example of Fig. 12 rules out preferred prudent and complete prudent semantics. In fact the sets of preferred prudent and complete prudent extensions of AF include $\{\alpha, \gamma\}$ and $\{\delta\}$. Given $\mathcal{RES}(\mathrm{AF}) = \{\mathrm{AF}_1, \mathrm{AF}_2\}$, we note that there is a unique preferred prudent (and complete prudent) extension of AF_1, namely $\{\alpha, \delta\}$, and a unique preferred prudent (and complete prudent) extension of AF_2, namely $\{\alpha, \gamma\}$. Then the intersection of the extensions of the argumentation frameworks in $\mathcal{RES}(\mathrm{AF})$, namely $\{\alpha\}$, is not included in the intersection of the extensions of AF, which is the empty set.

7. Discussion and conclusions

We have provided several general criteria for principle-based evaluation of extension-based argumentation semantics and applied them to a significant range of both "traditional" and more recent approaches. As to our knowledge, in the literature there are no other examples of such a systematic and extensive assessment of existing semantics proposals. In fact, example-based and/or pairwise comparisons are most commonly found, as discussed in Section 1. It has to be remarked, however, that the issue of formulating general principles for argumentation semantics is receiving an increasing attention in very recent years. Caminada and Amgoud [11] have discussed the issue of defining general *rationality postulates* for argumentation. In particular, the postulates of closeness under strict rules and consistency of extensions have been introduced, showing that there are argumentation systems where they are violated. Since these postulates are defined at the level of the language where arguments are explicitly constructed while our analysis lies at a more abstract level, this kind of work, while sharing the same spirit, represents a complementary and possibly synergic research line with respect to ours.

At a more specific level, relationships with some of the criteria we have introduced can be found in the literature. Defense related properties, namely admissibility and reinstatement, have often been used to compare argumentation semantics, at least at an informal level. We have provided a systematic formal characterization of these notions, including stronger and weaker versions, which, as to our knowledge, have not been considered before in the literature. In a similar line, Caminada has recently tackled the issue of providing postulates for the notion of reinstatement in the context of labellings of argumentation frameworks [10]. It turns out that the basic postulates on labeling provide an equivalent characterization of complete extensions, while adding other minimality/maximality requirements one may obtain a characterization of other Dung's semantics. While in the present paper we are dealing with extension-based rather than labeling-based argumentation semantics, extending the definition of the criteria we propose to the case of labeling-based semantics represents an interesting issue for future work. As to the directionality property, the related criterion of relevance has been proposed in [9] when discussing the differences between stable and semi-stable semantics. Given an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ an argument α is relevant with respect to β if and only if there is an undirected path from α to β in AF. Actually relevance is an equivalence relation, whose classes coincide with the partition of the (undirected) defeat graph into its connected components. A semantics satisfies the relevance criterion if the state of an argument is not affected by the presence of irrelevant arguments. Relevance can be regarded as a weaker form of the directionality criterion where only the existence but not the direction of attack relations is taken into account. As to adequacy criteria, starting from an analysis of hierarchical argumentation frameworks Modgil [18] has proposed, as already mentioned in Section 4.4, a general requirement for argumentation semantics, based on the notion of resolution of an argumentation framework and expressed as an if-and-only-if condition. We have shown that one of the directions of the condition is a special case of the criterion of skepticism adequacy, previously proposed in [3] and extended here, while we have generalized the other direction of the condition introducing the criterion of resolution adequacy.

From the application of evaluation criteria it has emerged that, with different degrees, every literature semantics violates some of the proposed criteria. Not all these violations are undesirable by themselves. For instance, some of them may be ascribed to the fact that some criteria are not strictly required (as in the case of I-maximality) or are too demanding (as in the case of strong admissibility). Other violations derive from deliberate design choices in order to achieve some desired behavior: this is the case, for instance, of $CF2$ semantics where admissibility and reinstatement are given up to achieve a "symmetric" treatment of odd- and even-length cycles. Grounded semantics is the closest to the goal of satisfying all the criteria, as it only violates the criterion of resolution adequacy in its various forms. At a general level, the situation sketched above stresses the problem of identifying which criteria are more suitable for different application domains and/or reasoning styles. A recent example of this kind of investigation is provided by the analysis carried out in [21] where it is suggested that in certain contexts epistemic reasoning is skeptical while practical reasoning is credulous. At a more technical level, one may raise the question whether the proposed criteria are incompatible altogether. Answering this question will be the subject of future work.

Acknowledgements

We are indebted to the anonymous referees and to the editors of this special issue for their helpful comments and suggestions.

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